

Limits and the Indeterminate Forms $0 \cdot \infty$, $\infty - \infty$, ∞^0 , 1^0 , 1^∞ , 0^∞ and 0^0

Limits and Indeterminate Forms: $0 \cdot \infty$ and $\infty - \infty$

Indeterminate forms like $0 \cdot \infty$ and $\infty - \infty$ can often be handled by reducing them algebraically to $0/0$ or ∞/∞ . Then L'Hospital's Rule can be applied.

Examples

Find $\lim_{x \rightarrow 0^+} x \ln(x)$.

Find $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{\sin(x)} \right)$.

Limits and Indeterminate Forms: ∞^0 , 1^0 , 1^∞ , 0^∞ and 0^0 .

These indeterminate forms require another approach. The following proposition is often helpful.

PROPOSITION:

If $f(x) > 0$, then $f(x)^{g(x)} = e^{\ln(f(x)^{g(x)})} = e^{g(x)\ln(f(x))}$.

In this case, $\lim_{x \rightarrow a} f(x)^{g(x)} = \lim_{x \rightarrow a} e^{g(x)\ln(f(x))} = e^{\lim_{x \rightarrow a} g(x)\ln(f(x))}$.

Examples

Evaluate $\lim_{x \rightarrow 0^+} (\cos x)^{1/x}$.

Evaluate $\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^x$.

Limit Problems

Using techniques you have learned in this course, find the following limits if they exist. (DO YOUR WORK ON A SEPARATE SHEET OF PAPER.) Decide first if an expression is an indeterminate form, and if so, of what kind. Check your work carefully. (You may also wish to use graphing or *Derive* to check your work.)

1.
$$\lim_{x \rightarrow 2} \frac{3x^2 - x - 2}{x^2 - 1}$$

2.
$$\lim_{x \rightarrow 1} \frac{3x^2 - x - 2}{x^2 - 1}$$

3.
$$\lim_{x \rightarrow 1^+} \frac{3x^2 - x - 1}{x - 1}$$

4.
$$\lim_{x \rightarrow 1} \frac{3x^2 - x - 1}{x - 1}$$

5.
$$\lim_{x \rightarrow \infty} \frac{3x^2 - x - 2}{x^5 - x}$$

6.
$$\lim_{x \rightarrow \infty} \frac{3x^4 - x + 1}{x^2 + 1}$$

7.
$$\lim_{x \rightarrow 0} \frac{\sin(5x)}{\sin(7x)}$$

8.
$$\lim_{x \rightarrow \infty} x \sin(1/x)$$

9.
$$\lim_{x \rightarrow \infty} 10 - 5x + x^2$$

10.
$$\lim_{x \rightarrow \infty} \frac{\ln(1+x)}{x}$$

11.
$$\lim_{x \rightarrow 0^+} x^x$$

12.
$$\lim_{x \rightarrow 0^+} \left(\frac{1+x}{1-x} \right)^{\frac{1}{x}}$$