Related Rates Problems: An Application of Implicit Differentiation and the Chain Rule

An interesting application of the Chain Rule and Implicit Differentiation occurs in problems involving *related rates*. In these problems, you might not have functions given explicitly, but now you typically have two functions whose rates of change are somehow related. For example, a related rates problem allows one to answer questions such as: "When a skier's blood vessels contract because of the cold, how fast is the velocity of the blood changing?" (See the Homework Supplement! Examples 1 and 2 are taken from *Calculus for the Life Sciences* by Greenwell, Ritchey and Lial; Example 3 is taken from Dr. Ching at John Hopkins University)

Example 1. For our first example, consider a rock dropped into a lake. Circular ripples spread over the surface of the water, with the radius of each circle increasing at the rate of 3/2 feet per second. Find the rate of change of the area inside the circle formed by a ripple at the instant the radius is 4 feet.

1. What do we want to know?

2. What is an expression relating the area of a ripple circle and the radius of a ripple circle. (Hint: remember to include what the area and radius depend on.)

3. What is the rate that we want to know? What should its units be? And when do we want to know it?

4. Find an expression for the quantity in (3.) and evaluate it. How can the relevant units guide us?

Question: Why can't we plug in the value, $r = 4$ in (2.) and proceed from there?

Example 2. A cone-shaped icicle is dripping from the roof. The radius of the icicle is decreasing at a rate of 0.2 cm/hour, while the length is increasing at a rate of 0.8 cm/hour. If the icicle is currently 4 cm in radius and 20 cm long, is the volume of the icicle increasing or decreasing, and at what rate?

1. What do we want to know?

2. What is an expression relating the volume of the cone to the radius we know and its length?

3. What is the rate that we want to know? What should its units be? And when do we want to know it?

4. Find an expression for the quantity in (3.) and evaluate it. How can the relevant units guide us?

Example 3. Recall the population model we had for hares and foxes in a past homework assignment. There exists a different kind of model for the populations of wolves (W) and bears (B) in a forest; their populations at any time t satisy:

$$W + B^3 = K,$$

where K is a constant. At a given time there are 10 bears and 1000 wolves. The bear population is increasing at approximately 1 bear per year. How fast is the wolf population increasing at that time?

1. What do we want to know?

2. What is an expression relating ?

3. What is the rate that we want to know? What should its units be? And when do we want to know it?

4. Find an expression for the quantity in (3.) and evaluate it. How can the relevant units guide us?