Elementary Derivatives and Rules of Differentiation

We have already seen some limits as well as some derivatives. Today we want to see more derivatives, but before we do so, we will look at some useful limits.

Some More Useful Limits

$\mathbf{1}^{st}$ **useful limit**: $\displaystyle\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta}=0$

Check:

$$\begin{aligned}
\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta} &= \lim_{\theta\to 0}\left(\frac{1-\cos\theta}{\theta}\right)\left(\frac{1+\cos\theta}{1+\cos\theta}\right)\\
&= \lim_{\theta\to 0}\frac{1-\cos^2\theta}{\theta\,(1+\cos\theta)}\\
&= \lim_{\theta\to 0}\frac{\sin^2\theta}{\theta\,(1+\cos\theta)}\\
&= \lim_{\theta\to 0}\left[\left(\frac{\sin\theta}{\theta}\right)\left(\frac{\sin\theta}{1+\cos\theta}\right)\right]\\
&= 1\cdot\left(\frac{0}{1+1}\right)=0.
\end{aligned}$$

$\mathbf{2}^{nd}$ **useful limit**: $\displaystyle\lim_{h\to 0}\frac{b^h-1}{h}=\ln(b),$ positive constant b.

Check this for yourself. You can do so with your calculator by picking a positive value for b, and graphing

$$\frac{b^x-1}{x}.$$

This function has a removable discontinuity where $x=0$. Look at the value (on the y-axis) that this function approaches as $x\to 0$, and compare it with $\ln b$.

Derivatives of More Elementary Functions

In addition to visiting some useful limits, we will need to refer to some of our old trigonometric identity friends. Use the following facts to help you determine the derivatives below:

$$\cos(x+h) = \cos(x)\cos(h) - \sin(x)\sin(h), \qquad \sin(x+h) = \sin(x)\cos(h) + \cos(x)\sin(h),$$

$$\lim_{\theta\to 0} \frac{\sin(\theta)}{\theta} = 1, \qquad \lim_{\theta\to 0} \frac{\cos(\theta)-1}{\theta} = 0, \qquad \lim_{h\to 0} \frac{b^h - 1}{h} = \ln(b), \qquad \text{positive constant } b.$$

1. Use the definition of the derivative to find a formula for $f'(x)$, where $f(x) = b^x$, and $b > 0$ is a given positive constant.

2. Use the definition of the derivative to find a formula for $g'(x)$, where $g(x) = \cos(x)$.

3. Use the definition of the derivative to find a formula for $p'(x)$, where $p(x) = \sin(x)$.

Derivatives of Some Elementary Functions

If you do not know the derivatives of the following elementary functions already, find them in your texts and record them here. We have not yet derived all of these results from the definition of the derivative, but we have done enough of this sort of work to remind you where these formulas come from. You should memorize these derivatives if you do not already know them and be able to derive them from the definition of the derivative.

$f(x)$	$f'(x)$
c, a constant	
x^r, constant r	
$\sin(x)$	
$\cos(x)$	
$\tan(x)$	
b^x, positive constant b	
e^x	
$\ln(x)$	

Some Limits You Should Know

$$\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta}=1, \qquad \lim_{\theta\to 0}\frac{\cos(\theta)-1}{\theta}=0, \qquad \lim_{h\to 0}\frac{b^h-1}{h}=\ln(b), \qquad \text{positive constant } b.$$

Practice with Differentiation

Combining the derivatives of the elementary functions on the previous page with the algebraic rules for derivatives on the page before, we can now find the derivative of any function that is a sum of elementary functions.

Find the following derivatives:

If $f(x) = x^2 + e^x$, then $f'(x) =$

If $g(x) = x^r + e^2$, where r is a constant, then $g'(x) =$

If $k(x) = 4 \cdot 100^2 + \ln(45)$, then $k'(x) =$

If $h(x) = \cos(x) + b^x$, where b is a positive constant, then $h'(x) =$

If $s(x) = \tan(x) + \ln x$, then $s'(x) =$