Slope Fields and Euler's Method

Slope Fields

DEFINITION: A **slope field** for a rate equation of the form

$$y'(t) = F(t, y(t))$$

consists of a set of (t, y) coordinate axes together with little sloped line segments placed at regularly spaced points in the coordinate plane. The slope of the line segment centered on a point with coordinates (t, y) has the numerical value $F(t, y(t))$.

A slope field for a rate equation is a useful way to visualize the information provided by the rate equation.

Example

1. Consider the rate equation $y'(t) = \frac{1}{4}t$. Complete the table below, then use it to sketch a slope field for this rate equation:

t:	0	0	0	1	1	1	2	2	2	3	3	3	4	4	4
y:	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2
$y'(t)$:															

2. Match the slope fields below with one of the two following rate equations. Explain your choice. (See *H-H*, Section 10.2, p. 495.)

A: $y'(x) = 2x$ B: $y'(x) = y$

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A slope field for a rate equation can help you visualize solutions to the rate equation. It can also help you visualize how Euler's Method is approximating solutions to the rate equation.

Example

Consider the initial value problem: $y'(t) = \frac{1}{4}t, \quad y(0) = 0.$

3. Check that the solution to this initial value problem has the formula $y(t) = \frac{1}{8}t^2$. Complete the following table, then plot a graph of this function on the slope field you constructed in 1. above.

t:	0	1	2	3	4
$y(t)$:					

4. Using Euler's Method, complete the following table to find a piecewise linear function $Y(t)$ approximating the solution of this same initial value problem, $y'(t) = \frac{1}{4}t$, $y(0) = 0$. Use a stepsize of $\Delta t = 2$. Plot this approximation on the same slope field in 1. above.

t	$Y(t + \Delta t) = Y(t) + \Delta Y$	slope, $m = \frac{1}{4}t$	$\Delta Y = m * \Delta t$
0	0		
2			
4			

5. Repeat problem 4., but this time use a stepsize $\Delta t = 1$. Plot the approximation $Z(t)$ on the same slope field in 1. above.

t	$Z(t + \Delta t) = Z(t) + \Delta Z$	slope, $m = \frac{1}{4}t$	$\Delta Z = m * \Delta t$
0	0		
1			
2			
3			
4			

Solutions to Rate Equations and Euler's Method

1. What is meant by a solution of a rate equation?
   Example 1: $y'(t) = 1$

   Example 2: $y'(t) = -y(t)$

2. What does a rate equation tell you about its solutions?

3. If you have a slope field, how can you graphically approximate a solution passing through a given point?

4. If you have a rate equation, how can you calculate a piecewise linear approximation to a solution passing through a given point?