Variables, Formulas, Functions, Graphs and Linear Approximations

Area of a Circle

$$A = \pi r^2, \qquad \text{area } A, \text{ radius } r.$$

Boyle's Law (1662)

$$PV = k, \quad \text{constant}$$

where P is the pressure, V the volume of a fixed amount of an ideal gas at a constant temperature. (... if the pressure increases, the volume decreases...)

Snow Tree Cricket Chirp Rate

$$R = 4T - 160$$

R is the chirps per minute of a snow tree cricket at temperature T degrees Fahrenheit.

Limitations of Formulas

1. What important information about its variables does a formula *not* provide?

2. Give an example of quantities whose relationship is difficult to express with a formula. How might this relationship be expressed?

The concept of a *function* was developed, in part, to overcome some of the limitations of formulas.

Definition (Function)

A *function* consists of a *domain* (the set of input values), a *range* (the set of output values), and a *rule* assigning a *unique* output value to *each* input value.

Example

3. For another one of the example formulas, write a function using complete notation.

Two well-defined functions are *equal* **if and only if they have the same domain and the same rule associating elements of the domain with elements of the range,** *even if they have different names and target sets, or the rule is expressed differently.*

Graph of a Function

The *graph* of the function $f(x)$ is the set of all ordered pairs $(x, f(x))$, where x varies over the domain. A graph can be sketched on a Cartesian coordinate plane.

5. Explain how the graph of a function specifies the rule of the function.

6. Plot a set of points which is *not* the graph of a function and explain why it is not.

Different ways to view a function

There are a number of different ways to view, or represent, a function. It is possible to think of the *same* function as:

a **table** a **formula**

a **graph** an **object**

a **domain and rule** a **machine**

It is important to be comfortable with the different ways we can view functions when we are considering functions in this course. If some of these ways make less sense than others, let us know! Considering multiple perspectives of the concept of function can help different students with different learning styles comprehend the concept in their own way. Different perspectives can also be useful in solving different problems.