MATH 114: Calculus 1 (Experienced) Quiz 7
Fall 2005

Lab Time: BUCKMORE **Your Name:**

This quiz is designed to illuminate your understanding of indeterminate forms, L'Hôpital's Rule, "dominant" functions and infinite limits.

a. *(5 points.)* Evaluate $\lim_{x\to\infty} \frac{(\ln(x))^3}{x}$. Explain your answer.

$$\lim_{x\to\infty} \frac{(\ln(x))^3}{x} \overset{L'H}{=} \lim_{x\to\infty} \frac{3\cdot(\ln x)^2\cdot\frac{1}{x}}{1} = \text{"}\infty\cdot 0\text{"} = \lim_{x\to\infty} \frac{3(\ln x)^2}{x}$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{6\ln x}{x} \overset{L'H}{=} \lim_{x\to} \frac{6}{x} = 0$$

Using L'Hôpital's Rule, one can see that $x\to\infty$ faster than $(\ln x)^3\to\infty$ ~~since~~ because x is dominant over $\ln x$ as $x\to\infty$

b. *(5 points.)* Evaluate $\lim_{x\to\infty} \frac{(\ln(x))^{1000000}}{x}$. Explain your answer.

$$\lim_{x\to\infty} \frac{(\ln x)^{100000}}{x} = 0.$$

Clearly if one uses L'Hopital's Rule ONE MILLION times will produce $\lim_{x\to\infty} \frac{1000000!}{x} = 0$. One could also say that since as $x\to\infty$, x is the dominant function, $\frac{(\ln x)^{100000}}{x} \to 0$.

c. **BONUS** *(5 points.)* Evaluate $\lim_{x\to\infty} \frac{(\ln(x))^m}{x}$ where m is **any** real number. Explain how (or if) the value of the limit depends on the values of m.

If $m=0$ $\lim_{x\to\infty} \frac{(\ln x)^m}{x} = \lim_{x\to\infty} \frac{1}{x} = 0$ (NOT INDETERMINATE)

If $m>0$ $\lim_{x\to\infty} \frac{(\ln x)^m}{x} = 0$ since x is dominant to $\ln x$ $x\to\infty$ faster than $\ln x\to\infty$

If $m<0$ $\lim_{x\to\infty} \frac{(\ln x)^m}{x} = \lim_{x\to\infty} \frac{1}{(\ln x)^{-m}x} = 0$ (NOT INDETERMINATE)

The value of the limit is zero regardless of the value m