MATH 114: Calculus 1 (Experienced)
Fall 2005

Quiz 3

Lab Time: **Your Name:** BUCKMIRE

We will consider the following rate equations and initial conditions on the interval $[0, 1]$.

$$S' = C, \quad C' = -S$$

$$S(0) = 0, \quad C(0) = 1$$

a. *(6 points.)* Use Euler's method with $\Delta t = 0.5$ to estimate the solutions $S(t)$ and $C(t)$ of the above initial value problem. Fill in the following table, which will help you find the approximating functions $\tilde{S}(t)$ and $\tilde{C}(t)$, which approximate the functions $S(t)$ and $C(t)$ which exactly solve the IVP.

$\Delta S \approx S' \cdot \Delta t$

$\Delta C \approx C' \cdot \Delta t$

$S_{new} = S_{old} + \Delta S$

$C_{new} = C_{old} + \Delta C$

t	S	C	S'	C'	ΔS	ΔC
0	0	1	1	0	0.5	0
$\frac{1}{2}$	0.5	1	1	-0.5	0.5	-0.25
1	1	0.75	XXXXXX XXXXXX XXXXXX	XXXXXX XXXXXX XXXXXX	XXXXXX XXXXXX XXXXXX	XXXXXX XXXXXX XXXXXX

Note: You should not need to use a calculator for this problem, but if you must use your calculator, DO NOT round off any decimal points.

b. *(4 points.)* Show that $S(t) = \sin(t)$, $C(t) = \cos(t)$ are the exact solutions to the IVP.

$t = 0$, $S(0) = \sin(0) = 0$ ✓ IC

$t = 0$, $C(0) = \cos(0) = 1$ ✓ IC

$(\sin(t))' = \cos t \iff S' = C$ ✓ RE

$(\cos t)' = -\sin t \iff C' = -S$ ✓ RE

c. **BONUS** *(5 points.)* Show that, from the IVP alone, we can tell that the functions $S(t)$ and $C(t)$ obey the expression $S^2 + C^2 = 1$ for every value of t. (HINT: Differentiate this expression with respect to t and use *all* the information from the initial value problem.)

$S^2 + C^2 = 1$

When $t = 0$, $S(0) = 0$, $C(0) = 1$

$0^2 + 1^2 = 1$ ✓

The expression is true at $t = 0$.

Suppose $S^2 + C^2 = 1$

Differentiate both sides respect to t

$\frac{d}{dt}(S(t))^2 + \frac{d}{dt}(C^2(t)) \stackrel{?}{=} \frac{d}{dt}(1)$

Using Chain Rule

$2S(t) \cdot S'(t)$

$+ 2C(t) \cdot C'(t)$

$\stackrel{?}{=} 0$

$2S \cdot C + 2C \cdot (-S) \stackrel{?}{=} 0$

$2SC - 2CS \stackrel{?}{=} 0$

$0 = 0$ ✓