QUIZ 1 SOLUTIONS:

① A linear transformation is defined as any function f satisfying $f(a+b) = f(a) + f(b)$ and $f(cx) = cf(x)$, where a, b and c are unknown parameters. Show that, according to this definition $f(x) = 3x + 2$ is NOT a linear transformation, but $f(x) = 3x$ is.

(i) $f(x) = 3x + 2$ is NOT a linear transformation:

$$f(a+b) = 3(a+b) + 2 = 3a + 3b + 2 \qquad (A)$$

$$f(a) + f(b) = 3a + 2 + 3b + 2 = 3a + 3b + 4 \qquad (B)$$

Since $(A) \neq (B)$, $f(x) = 3x + 2$ is NOT a linear transformation. ∎

(ii) $f(x) = 3x$ is a linear transformation.

$$f(a+b) = 3(a+b) = 3a + 3b \qquad (A)$$

$$f(a) + f(b) = 3a + 3b \qquad (B)$$

Since $(A) = (B)$, the 1st condition is met; $f(a+b) = f(a) + f(b)$. Now we check the 2nd condition.

$$f(cx) = 3(cx) = 3cx \qquad (\hat{A})$$

$$c \cdot f(x) = c \cdot 3(x) = c \cdot 3x = 3cx \qquad (\hat{B})$$

$(\hat{A}) = (\hat{B})$, So $f(cx) = c \cdot f(x)$. Since $f(x) = 3x$ meets both conditions, $f(x) = 3x$ is a linear transformation.