# SHOW ALL YOUR WORK

Given the function $f(x) = e^{\sqrt{x}}$

a) (*4 points.*) Find the function $g$ which is the inverse of $f(x)$

$f(x) = y = e^{\sqrt{x}}$

Switch x and y and solve for y

$x = e^{\sqrt{y}}$

$\ln(x) = \ln(e^{\sqrt{y}})$

$\ln(x) = \sqrt{y} \ln e$

$\ln(x) = \sqrt{y}$

$y = \boxed{(\ln(x))^2 = g(x)}$

b) (*1 point.*) Find the number **a** which solves the equation $f(a) = 2$. (Please, no decimal points!)

$f(a) = 2 \Leftrightarrow f^{-1}(f(a)) = f^{-1}(2)$

$a = f^{-1}(2)$

$a = g(2) = \boxed{(\ln 2)^2 = a}$

c) (*1 point.*) Find the number **b** which solves the equation $g(b) = 0$. (Please, no decimal points!)

$g(b) = 0$

$b = g^{-1}(0) = f(0) = e^{\sqrt{0}} = e^0 = \boxed{1 = b}$

d) (*2 points.*) Compute $g'(2)$ directly from the derivative of $g$. (Please, no decimal points!)

$g'(x) = 2 \cdot \ln(x) \cdot \frac{1}{x}$

$g'(2) = 2 \cdot \ln 2 \cdot \frac{1}{2}$

$\boxed{g'(2) = \ln(2)}$

e) (*2 points.*) Find $f'(a)$ where **a** is the solution of $f(a) = 2$ from part **b.** [HINT: It is probably easier for you to use your answer to part **(d)** than differentiating $f(x)$ abd evaluating at **a**.]

$f'(a) = \frac{1}{g'(f(a))} = \frac{1}{g'(2)} = \frac{1}{\ln 2}$

$f(x) = e^{\sqrt{x}}$

$f'(x) = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

$f'((\ln 2)^2) = e^{\sqrt{(\ln 2)^2}} \cdot \frac{1}{2\sqrt{(\ln 2)^2}}$