Math 114 *Fall 2005*

Lab 5: Applications of Implicit Differentiation and the Chain Rule

Name:

Lab Day and Time:

In this lab you will work on related rates problems. The first two are similar to those you have seen in class and in the homework. The last will appear in an assignment you have been given from Hocklead-Tarmin. Their request is attached.

Problem 1: Allometric equations describe the scaling relationship between two measurements. Icthyosaurs are a group of marine reptiles that were fish-shaped and comparable in size to dolphins. They became extinct during the Cretaceous period. based on a study of twenty fossil skeletons, it was found that skull length (in cm) and backbone length (in cm) of an individual were related through the allometric equation

$$[\text{skull length}] = 1.162[\text{backbone length}]^{0.933}.$$

How is the growth rate of the backbone related to the growth rate of the skull?

Problem 2: Consider a case of a parcel of air rising quickly in the atmosphere; it expands without exchanging heat with the surrounding air. According to physics, the volume (V) and the temperature (T) of the parcel of air are related according the to following equation:

$$TV^{\gamma-1} = C$$

where γ is a constant and $\gamma \approx 1.4$ for sufficiently dry air, and C is a constant. The temperature is measured in the Kelvin (or absolute temperature) scale. Since rising air expands, the volume of the parcel of air increases with time; i.e. $dV/dt > 0$, where t denotes time. How does the temperature of the air parcel change as it rises? How would the temperature of the air parcel change if it was falling and the volume was therefore decreasing?

For this part of the lab, write up these problems as if you were doing it for a textbook example. That is, the presentation of the problems should be clear and include english words, phrases and sentences. You want the students who read your examples to understand related rates as well as possible!

Part II

Hocklead-Tarmin, Inc.
15000 Dalame Dr.
Verden, DO 28802-0540
October 18, 2005

Independent Mathematical Contractors, Inc.
Suite 2, Strawmarket Business Plaza
Ockeagler, AC π0041

Dear IMC:

As a primary contractor in defense and the aerospace industry, Hocklead-Tarmin maintains several industrial sites in and around Verden, Docolora. In the course of expanding one of these, we find it expedient to add a number of free standing vertical cylindrical fuel tanks, as shown in figure 1. These are filled and discharged through the valve at the base of tank. As the tanks are large (20' high with radii of 8'), there is a clear incentive to be sure that should they develop a leak the fuel release would be adequately contained, and we are therefore building a wall around each tank that will catch the fuel in this case. Simultaneously, we would like to avoid spending too much on a very large wall if a small one will suffice, and we are therefore contacting you to obtain an estimate for the amount of fuel that might be released. Figure 1: Fuel tank The greatest risk of unintended discharge is when the tank is being filled or emptied through the valve (the diameter of which is approximately 9"), as through unintended contact or mechanical failure the valve could fail and allow unimpeded discharge through the pipe to which the valve is connected. In this case, it is known that the rate at which the height of the fuel in the tank will change is directly proportional to the ratio of the square of the diameter of the valve to the square of the diameter of the tank as well as the square root of the height of the liquid in the tank. The constant of proportionality is $k = (2g)^{1/2}$ (where g is the acceleration due to gravity; g = 9.8 m/s^2).

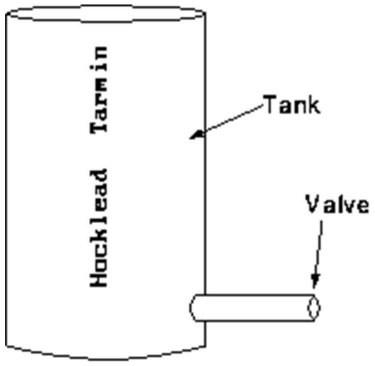

Figure 1: Tank

Owing to our strong accident-containment procedures, any spill in these circumstances should be stopped within 10 minutes of its initiation. We need to know under these conditions how much fuel will need to be contained if the leak starts with a full tank and proceeds for either 5 or 10 minutes.

In order to push the industrial site into production as soon as possible, we need your 2–3 page report by the 31st of October (Monday labs) or the 1st of November (Tuesday labs). Please attach any relevant figures to explain your calculations to us. We are very busy; so although we definitely need to understand your conclusions, we need to do so in the most efficient way possible. After careful consideration, we have also contracted with the redoubtable Drs. Buckmire and Gallegos, mathematicians in your area, to field any questions you may have in the course of your work.

We look forward to hearing from you.

Sincerely,

O.W.R. Eit III CEO,

Hocklead-Tarmin, Inc.