HOMEWORK 7 SOLUTIONS:

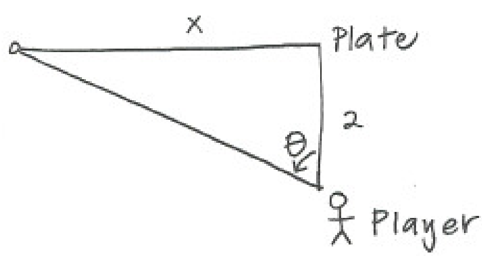
SM 2.8: 29, 30, 51 ; Class 18 Handout

SM 6.2: 20, 24, 35, 36

SM 6.7: 5, 6

Sec. 2.8: 29, 30, 51

29. figure:



We want to know $\frac{d\theta}{dt}$ when $x = 0$ and $\frac{dx}{dt} = -130$ ft/s.

An equation relating x and θ is:

$$\tan\theta = \frac{x}{2}$$

Differentiating both sides: $\frac{d}{dt}\tan\theta = \frac{d}{dt}\frac{x}{2}$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{2}\frac{dx}{dt}$$

using $\frac{1}{\sec^2\theta} = \cos^2\theta$ $\qquad \frac{d\theta}{dt} = \frac{1}{2}\frac{dx}{dt} \cdot \cos^2\theta$

* when $x = 0$, $\theta = 0 \Rightarrow \cos^2 0 = 1$. So,

using the relevant values: $\frac{d\theta}{dt} = \frac{1}{2}(-130)\,1 = -65$ rad/s

$\boxed{\text{The player's eyes must move at a rate of } -65 \text{ rad/s.}}$

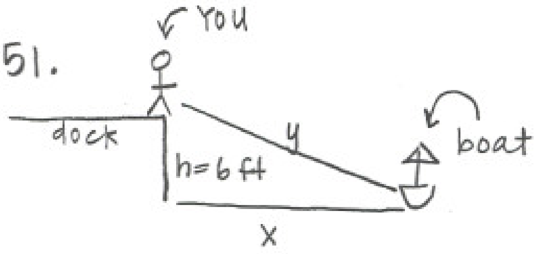
30. We are in the same situation as problem 29, but now we're given $\frac{d\theta}{dt} = \frac{\pi}{2}$ and we want the corresponding $\frac{dx}{dt}$.

So: $\frac{d\theta}{dt} = \frac{1}{2}\frac{dx}{dt}$ (from 29)

$$\frac{\pi}{2} = \frac{1}{2}\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \pi \text{ ft/s}$$

The fastest pitch you could watch cross home plate (while maintaining focus!) moves at π ft/s.

51.



We know: $\frac{dy}{dt} = 2$ ft/s

$h = 6$ ft (and is constant)

We want to know $\frac{dx}{dt}$ when $x = 20$, $x = 10$.

A relationship between x and y and h is:

$x^2 + h^2 = y^2$ (pythagorean's theorem)

$2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$ since h is constant.

when $x = 20$ we need y: $20^2 + 6^2 = y^2$

$436 = y^2 \Rightarrow y \approx 20.88$ ft

when $x = 10$ we need y: $10^2 + 6^2 = y^2$

$136 = y^2 \Rightarrow y \approx 11.66$ ft.

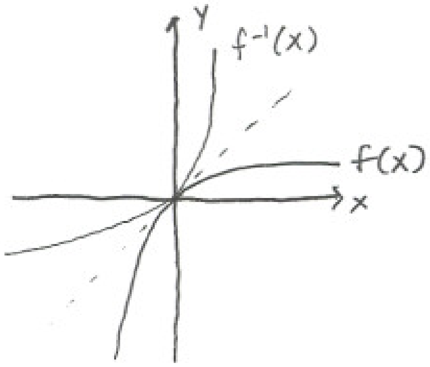
When $x = 20$:

$$2(20)\frac{dx}{dt} = 2(20.88)\cdot 2$$

$$\frac{dx}{dt} = 2.088 \text{ ft/s}$$

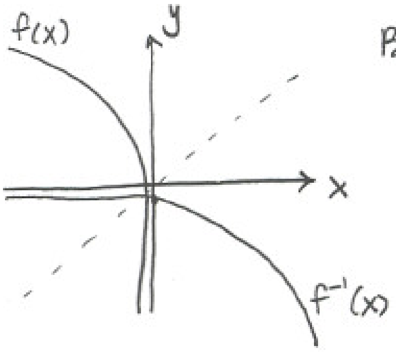
At 20 ft away from the dock, the boat's speed is ≈ 2.088 ft/s

35.



Because the inverse function is the reflection of $f(x)$ across the line $x=y$, it will be concave up.

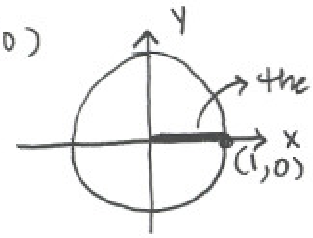
36.



Because the inverse function is the reflection of $f(x)$ across the line $x=y$, it will be concave down.

6.7: 5,6

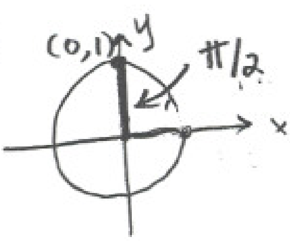
5. $\sin^{-1}(0)$



$\Rightarrow \sin(0)=0$

$\boxed{\sin^{-1}(0)=0}$

6. $\cos^{-1}(0)$



$\Rightarrow \cos(\pi/2)=0$

$\boxed{\cos^{-1}(0)=\pi/2}$

When $x = 10$:

$$2(10)\frac{dx}{dt} = 2(11.66)\cdot 2$$

$$\frac{dx}{dt} = 2.332 \text{ ft/s}$$

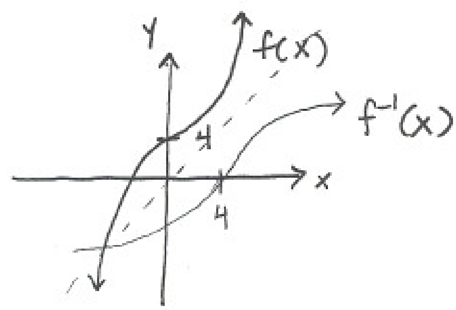
$\boxed{\text{At 10 ft. away from the dock, the boat's speed is} \approx 2.332 \text{ ft/s.}}$

SHEET SOLUTIONS AT END

SM Sec 6.2: 20, 24, 35, 36

20. $f(x) = x^5 + 4$:

The graph of $f(x)$ looks like:



The function is 1-1 (it passes the horizontal line test) So it has an inverse.

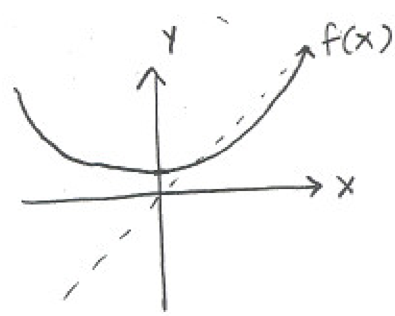
To solve for the inverse:

$$x = y^5 + 4$$

$$x - 4 = y^5 \Rightarrow \boxed{f^{-1}(x) = \sqrt[5]{x-4}}$$

24. $f(x) = \sqrt{x^2+1}$

The graph of $f(x)$ looks like:



The function is <u>not</u> 1 to 1

So it does NOT have an inverse on the domain of real numbers.

(If you restricted its domain it would)

Supplementary Related Rates Problems

Name:

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1. *From Calculus for the Life Sciences by Greenwell, Ritchey and Lial; Example 5.* Blood flows faster the closer it is to the center of a blood vessel because of the reduced friction with cell walls. According to Poiseuille's laws, the velocity V of blood is given by

$$V = k(R^2 - r^2),$$

where R is the radius of the blood vessel, r is the distance of a layer of blood flow from the center of the vessel, and k is a constant, assumed here to equal 375. Suppose a skier's blood vessel has radius $R = 0.08$ millimeter and that cold weather is causing the vessel to contract at a rate of $dR/dt = -0.01$ millimeter per minute. How fast is the velocity of the blood changing?

hint: treat r as constant!

We are interested in $\frac{dV}{dt}$ when $R = 0.08$, $\frac{dR}{dt} = -.01$ mm/min

Use the equation above and differentiate both sides:

$$\frac{dV}{dt} = k\left(2R\frac{dR}{dt} - 0\right)$$
$$= 2kR\frac{dR}{dt}$$

So, with our relevant values

$$\frac{dV}{dt} = 2(375)(0.08)(-.01)$$

$$\frac{dV}{dt} = -.6 \text{ mm/min}$$

$\boxed{\text{The velocity of the blood is decreasing at a rate of } -.6 \text{ mm/min.}}$

2. *From Calculus for the Life Sciences by Greenwell, Ritchey and Lial; Problem 17.* Sociologists have found that crime rates are influenced by temperature. In a midwestern town of 100,000 people, the crime rate has been approximated as

$$C = \frac{1}{10}(T - 60)^2 + 100,$$

where C is the number of crimes per month and T is the average monthly temperature in degrees Fahrenheit. The average temperature for May was 76°, and by the end of May the temperature was rising at the rate of 8° per month. How fast is the crime rate rising at the end of May?

We are interested in $\frac{dC}{dt}$ when $T = 76°$ and $\frac{dT}{dt} = 8°/\text{mo}$:

Using our equation above and differentiating both sides:

$$\frac{dC}{dt} = \frac{1}{10} \cdot 2(T-60)\frac{dT}{dt} + 0$$

$$= \frac{1}{5}(T-60)\frac{dT}{dt}$$

Using our relevant values:

$$\frac{dC}{dt} = \frac{1}{5}(76-60)(8°) = 25.6 \text{ crimes/month}$$

$\boxed{\text{The crime rate is increasing by 25.6 crimes/month.}}$