Test 1: Math 114

Calculus 1 (Experienced) Tuesday, September 27, 2005

Name: BUCKMIRE

Prof. Buckmire (10:30am)
Prof. Gallegos (1:30pm)
(circle one)

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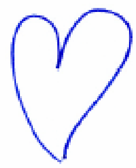
1. Read each question carefully and be sure to answer appropriately. When it makes sense, answer in complete sentences.
2. Partial credit will be given, but only if we can see the correct parts. So **show all of your work** and **explain your answers**!
3. Although the exam is designed to be completed in 55 minutes you may have from 7:00 to 9:00pm.
4. Before you hand in your exam please sign the pledge below.
5. Make sure your "Blue Notes" is attached.
6. Relax and enjoy... (Good Luck!)

Pledge: I, __________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

No.	Score	Maximum
1		1
2		33
3		33
4		33
BONUS		5
Total		**100**

1. The Joke Problem. Draw something that is not a yam.



2. The CSI Problem. Newton's Law of Cooling can be applied to a dead body at a crime scene: the body temperature cools at a rate proportional to the temperature difference between the body and the surrounding environment.

A murder was reported at the ScairEE Motel. The police arrived at 12:00am and immediately measured the body's temperature; they found it to be 33° C. They proceeded to collect evidence from the scene and finished at 1:15am. They again measured the temperature of the body, which had cooled to 30° C and left the scene. The temperature of the motel room was 23° C the entire time.

(a) Using the information from above, write down the IVP for the temperature of the body at any time, $T(t)$.

10

$$T' \propto T - A \Rightarrow T' = k(T - A)$$

$A = 23°C$

$$T' = k(T - 23)$$

$$T(0) = 33°$$

(b) Verify that $T(t) = 23 + 10e^{kt}$ is the solution to your IVP in **(a)**.

10

Check initial condition

$$T(0) = 23 + 10e^{k\cdot 0} = 23 + 10e^{0}$$
$$= 23 + 10 \cdot 1$$
$$= 23 + 10$$
$$= 33 ✓$$

Check rate equation

$$T' = \frac{d}{dt}(23 + 10e^{kt}) = 0 + 10ke^{kt}$$

$$k(T - A) = k[23 + 10e^{kt} - 23] = k[10e^{kt}]$$

So, $T' = k(T - A)$

(c) Estimate $T'(0)$ using the data measured at the scene.

$$T'(0) \approx \frac{T(75) - T(0)}{75 - 0} \approx \frac{30 - 33}{75} = -\frac{3}{75} = -\frac{1}{25}$$

$$= -0.04 \ {}^{\circ}C/min$$

5

(d) Use part (c) to determine an estimate for your constant of proportionality k in (a).

$$T'(0) = K(T(0) - 23)$$
$$-0.04 = K(33 - 23)$$
$$-0.04 = 10K$$
$$-0.004 = K$$

5

(e) The temperature data measured by the CSI team and the k estimate from (d) can be used to estimate time of death. In your opinion, do the 2 temperatures provided by the police allow you to make a good estimate of k? Use at least one sentence to explain why. If not, what could the CSI team do differently to improve the estimate?

3 The two temperatures are taken very far apart in time so that the estimate for $T'(0)$ is very inaccorate and thus leads to an very inaccorate value for K. To improve the estimate for K, the CSI team should wait MUCH LESS TIME than 75 minutes to measure the temperature of the body again.

3. The "Been There, Done That" Problem. Given the following SIR model, answer the questions below.

$$\begin{aligned} S'(t) &= -0.004S(t)I(t), \quad S(0) = 1000; & (1)\\ I'(t) &= 0.004S(t)I(t) - 0.05I(t), \quad I(0) = 30; & (2)\\ R'(t) &= 0.05I(t), \quad R(0) = 5. & (3) \end{aligned}$$

(a) How many days does this disease last?

The recovery period is 20 days.

6

$b = 0.05$

$\frac{1}{b} = \frac{1}{0.05} = 20$

(b) At time $t = 0$, is the disease spreading or dying out?

12

$a = 0.004$ $\quad S_* = \frac{b}{a} = \frac{0.05}{0.004} = 12.5 < 1000$

$b = 0.05$

Since $S(0) > S_*$ there is an epidemic $(I' > 0)$

$I'(0) = 0.004 \cdot S(0) \cdot I(0) - 0.05 I(0)$

$= 0.004 \cdot 1000 \cdot 30 - 0.05 \cdot 30$

$= 120 - 1.5$

$= 118.5 > 0$

(c) Circle one: Your answer to Part (b) indicates that S_0 $\underline{>}$ $S_\star$, the threshold value.

(i) >

(ii) <

3 (iii) =

(iv) it doesn't give any relevant information

(d) Find an equation for the tangent line to the graph of $I(t)$ at $t = 0$.

$I'(0) = 118.5$ $\qquad$ $I(t) \approx I(0) + I'(0)t$

$I(0) = 30$

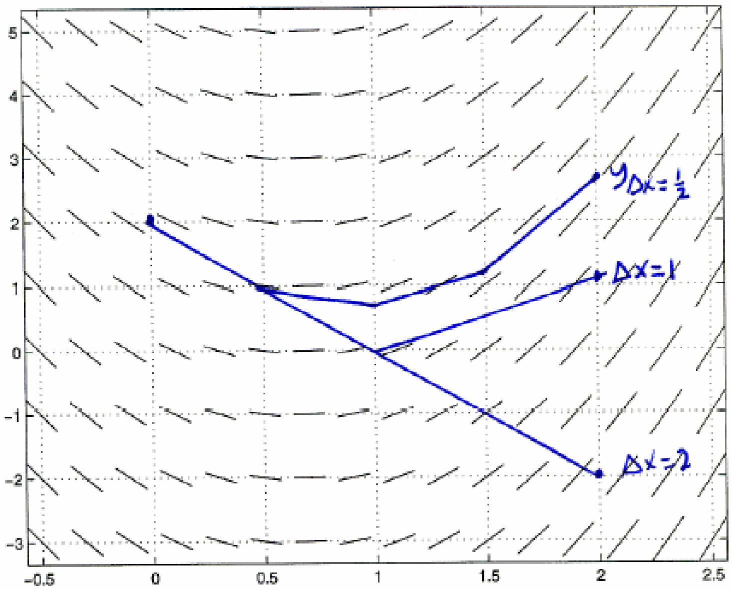
$y = 30 + 118.5t$

12

4. The Visual Problem. The following is a slope field for $y' = 3x - 2$.

(a) Use the slope field below to draw successive piecewise linear approximations to $y(2)$, one which uses $\Delta x = 2$, one with $\Delta x = 1$, and finally one with $\Delta x = .5$. Start at the initial value $(0, 2)$.

18



(b) Find successive approximations to the value $y(2)$ using Euler's Method with $\Delta x = 2$, and with $\Delta x = 1$ and then with $\Delta x = 0.5$.

$\Delta x = 2$

x	y	y'	Δy
0	2	-2	-4
2	-2		

$y(2) \approx -2$

15

$\Delta x = 1$

x	y	y'	Δy
0	2	-2	-2
1	0	1	1
2	1		

$y(2) \approx 1$

$\Delta x = 1/2$

x	y	y'	Δy
0	2	-2	-1
1/2	1	-1/2	-1/4
1	3/4	1	1/2
3/2	5/4	2.5	1.25
2	5/2		

$y(2) \approx 2.5$

(c) **BONUS.** Write a short essay explaining whether the exact value of $y(2)$ to the initial value problem $y' = 3x - 2, \qquad y(0) = 2$ is greater or less than the values you estimated in part (b).