Test 3: Math 114

Calculus 1 (Experienced) Tuesday, November 29, 2005

Name: ______________________________

Prof. Buckmire (10:30am)
Prof. Gallegos (1:30pm)
(CIRCLE ONE)

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0. **NO CALCULATORS** are to be used on this exam. Please refer to the sheet of helpful values on the last page.
1. Read each question carefully and be sure to answer appropriately and thoroughly. When it makes sense, **answer in complete sentences**.
2. Partial credit will be given, but only if we can see the correct parts. So **show all of your work** and **explain your answers**!
3. Although the exam is designed to be completed in 55 minutes you may have from 7:00 to 9:00pm.
4. Before you hand in your exam please sign the pledge below.
5. Make sure your "Blue Notes" is attached.
6. Relax and enjoy... (Good Luck!)

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

No.	Score	Maximum
1		1
2		30
3		19
4		30
5		20
BONUS		5
Total		**100**

1. Holidays! *(1 point)* Draw or name your favorite Thanksgiving food item. (Alternately you could name your favorite doughnut–it depends which you like better!)

2. Fun with Rate Equations. *(30 points.)*

a. For the rate equation

$$\frac{dx}{dt} = -5 + x^2,$$

find all equilibrium values, determine their stability and sketch representative solutions $x(t)$ for $t > 0$.

b. For the rate equation

$$\frac{dx}{dt} = 4 + x^2,$$

find all equilibrium values, determine their stability and sketch graphs of representative solutions $x(t)$ for $t > 0$.

c. The rate equations in (a) and (b) both have the form:

$$\frac{dx}{dt} = r + x^2.$$

The number of equilibrium values depends on the value of r. Find a value of r so that the rate equation only has **one** equilibrium value; determine the stability of this sole equilibrium value and sketch graphs of representative solutions $x(t)$ for $t > 0$.

3. Fun with Numbers. *(19 points.)*
In this problem, your goal is to answer the following question: **"For what positive number is the sum of its reciprocal and four times its square a minimum?"**

a. Give the magical number a name:

b. Write an expression for (i) the reciprocal of the magical number; (ii) four times the square of the magical number; (iii) the sum of (i) and (ii).

c. Use optimizing techniques to answer the question **in bold** at the top of the page.

4. Your turn to grade! *(30 points.)* In the following limit evaluations, find **any** and **all** mistakes. In the original work, label all errors numerically (i.e. 1, 2, etc.). Then, beneath the work, state why each error is indeed an error. Finally, evaluate the limit correctly. If there is no error, only explain the steps that were taken.

a.

$$\begin{aligned}
\lim_{x\to 0^+} \frac{\sin(x)}{x^3} - \frac{1}{x^2} &= \frac{\sin(x)}{x^3} - \frac{x}{x^3} \\
&= \lim_{x\to 0^+} \frac{\sin(x) - x}{x^3} \\
&= \lim_{x\to 0^+} \frac{\cos(x)}{3x^2} \\
&= \frac{-\sin(x)}{6x} \\
&= 0
\end{aligned}$$

b.

$$\begin{aligned}
\lim_{x\to\infty} x^{1/\sqrt{x}} &= \lim_{x\to\infty} e^{\ln(x^{1/\sqrt{x}})} \\
&= \lim_{x\to\infty} e^{\frac{1}{\sqrt{x}}} \cdot \ln(x) \\
&= \lim_{x\to\infty} e^{\frac{\ln(x)}{\sqrt{x}}} \\
&= e^{\lim_{x\to\infty} \frac{\ln(x)}{\sqrt{x}}} \\
&= e^{\lim_{x\to\infty} \frac{\frac{1}{x}}{0.5x^{-0.5}}} \\
&= e^{\lim_{x\to\infty} \frac{2x^{0.5}}{x}} \\
&= e^{\lim_{x\to\infty} \frac{2}{x^{0.5}}} \\
&= 1
\end{aligned}$$

5. The Multiple Choice You Have Always Wanted. *(20 points.)*
TRUE or FALSE – put your choice in the box. You will receive 4 times more credit for the explanation of your choice than the choice itself. Remember if you think a statement is TRUE you must prove it is ALWAYS true. If you think a statement is FALSE then all you have to do is show there exists a counterexample which proves the statement is FALSE.

(a) TRUE or FALSE? For a continuous function $f(x)$ which possesses first derivative $f'(x)$ and second derivative $f''(x)$, whenever $f''(x) = 0$ the graph of the curve $f(x)$ experiences a change in concavity.

(b) TRUE or FALSE? Every sequence of approximations $x_0, x_1, x_2, \ldots$ generated by using Newton's Method Recursive Algorithm to solve $f(x) = 0$ must converge to a value x^* such that $f(x^*) = 0$ for any initial guess x_0.

BONUS *(5 points)*: From the information presented in the following table for an unknown function $\phi(z)$, use Taylor polynomials to obtain your best estimate for $\phi(2)$.

z	$\phi(z)$	$\phi'(z)$	$\phi''(z)$
0	10	10	2