Math 114 Fall 2005

Report on Exam 3

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Point Distribution (N=62)

Range	100+	90+	80+	70+	60+	60-
Grade	A+	A	B	C	D	F
Frequency	0	9	17	16	11	9

Comments

Overall Way to go! You made it through your last regular exam before the final. The average score was 75.

#1 Holidays! We hope you enjoyed this problem. It made us hungry to grade. Some of you are creative in your drawings!

#2 Fun with rate equations. These rate equations are generic rate equations, but could be related to the population models and other various rate equations you saw in class. Remember that when dealing with a *rate equation* the derivative is with respect to time. Thus, when you look at the second derivative of the function, you have to use the chain rule. Also $t > 0$ does not mean $x(t) > 0$. If t is on the horizontal axis, then it simply means we don't need to consider solutions to the left of the axis. In the second part, when $r = 4$, there are **no** equilibrium solutions. In this case the derivative is positive. This means that solutions will always be increasing. It does not mean there are no solutions. Finally, in the third part, if $r = 0$ you will only have the equilibrium value $x^* = 0$.

#3 Fun with Numbers. This was a textbook optimization problem–literally from your textbook! Most of you successfully found the function $f(x) = 1/x + 4x^2$. *This* is the function you want to optimize, so you need to take its derivative and set it equal to zero to find its critical points. **Once you find the critical value, you need to verify it is a minimum!** You could use either the First Derivative Test ($f' < 0$ then $f' > 0$) or the Second Derivative Test ($f''(c) > 0$). The magic number in this case is 1/2.

#4 Your turn to grade! Notice that the instructions tell you to explain the steps if they are correct. This was one common error on the exam.

For part 1:

a.

$$\lim_{x\to 0^+} \frac{\sin(x)}{x^3} - \frac{1}{x^2} = \frac{\sin(x)}{x^3} - \frac{x}{x^3} \quad ; \text{ error: limit notation has been dropped.}$$

$$= \lim_{x\to 0^+} \frac{\sin(x) - x}{x^3} \quad ; \text{ no error in this step from the last.}$$

$$= \lim_{x\to 0^+} \frac{\cos(x)}{3x^2} \quad ; \text{ error: the derivative of the numerator was taken incorrectly}$$

$$= \frac{-\sin(x)}{6x} \quad ; \text{ error: previous step was not in indeterminate form (if it were correct).}$$

error: limit notation has been dropped again.

$$= 0 \quad ; \text{ error: if correct, the previous step was not in indeterminate form.}$$

error: $0/0 \neq 0$

The correct limit evaluation is as follows:

$$\begin{aligned}
\lim_{x\to 0^+} \frac{\sin(x)}{x^3} - \frac{1}{x^2} &= \frac{\sin(x)}{x^3} - \frac{x}{x^3} && ;\ \text{algebraic simplification (multiply by } x/x \text{ to get a common denominator)} \\
&= \lim_{x\to 0^+} \frac{\sin(x) - x}{x^3} && ;\ \text{again, algebraic simplification} \\
&= \lim_{x\to 0^+} \frac{\cos(x) - 1}{3x^2} && ;\ \text{previous form indeterminate (0/0) so L'Hopital's rule used.} \\
&= \lim_{x\to 0^+} \frac{-\sin(x)}{6x} && ;\ \text{previous form indeterminate (0/0) so L'Hopital's rule used.} \\
&= \lim_{x\to 0^+} \frac{-\cos(x)}{6} && ;\ \text{previous form indeterminate (0/0) so L'Hopital's rule used} \\
&= \frac{-1}{6} && ;\ \text{evaluation of the limit.}
\end{aligned}$$

For the second part there were no errors in the computation of the limit and evaluation of the indeterminate form.

b.

$$\lim_{x\to\infty} x^{1/\sqrt{x}} = \lim_{x\to\infty} e^{\ln(x^{1/\sqrt{x}})}$$

rewrite the indeterminate expression (∞^∞) using properties of logarithms.

$$= \lim_{x\to\infty} e^{\frac{1}{\sqrt{x}}\cdot \ln(x)}$$

properties of logarithms.

$$= \lim_{x\to\infty} e^{\frac{\ln(x)}{\sqrt{x}}}$$

algebraic simplification.

$$= e^{\lim_{x\to\infty} \frac{\ln(x)}{\sqrt{x}}}$$

properties of limits or continuity of exponential function

$$= e^{\lim_{x\to\infty} \frac{\frac{1}{x}}{0.5x^{-0.5}}}$$

previous limit expression indeterminate (∞/∞) so L'Hopital's rule used.

$$= e^{\lim_{x\to\infty} \frac{2x^{0.5}}{x}}$$

algebraic simplification.

$$= e^{\lim_{x\to\infty} \frac{2}{x^{0.5}}}$$

algebraic simplification.

$$= 1$$

limit evaluation.

#5 The multiple choice you have always wanted! We hope that you enjoyed your multiple choice question. Being the sneaky math professors we are, we gave you two statements that were false. In part (a) there are many functions who have places where the second derivative is equal to zero but have no change in concavity. Examples to consider are $f(x) = x^4$ or $f(x) = x$. (That last example is a bit sneaky–but it is true!) In part (b) the statement is that Newton's method always converges. As we saw in lab (and class) there are many ways that Newton's method can fail. A particular guess might have zero derivative (i.e. $f'(x_0) = 0$) or lead to a subsequent guess that has zero derivative. A guess might lead to a cycle ($x_0 = 1, x_1 = -1, x_2 = 1, x_3 = -1, \ldots$). These are all ways that Newton's method can fail.