Math 114 Fall 2005

Report on Exam 1

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Point Distribution (N=71)

Range	100+	92+	90+	87+	83+	80+	77+	73+	70+	67+	63+	60+	60-
Grade	A+	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Frequency	12	18	9	4	1	5	1	5	3	2	2	1	4

Comments

Overall More than half the class earned an A grade on the first exam. This is an excellent result!! On the other end of the spectrum, only 4 students failed the exam. The average score was an 85.

#1 The Joke Problem. We hope you enjoyed this problem. My, what good artists you are!

#2 The CSI Problem. This is a Newton's Law of Cooling modeling problem disguised as a CSI episode. **(a)** We were only looking for as much of the initial value problem as one knows at the time, i.e. $T' = k \cdot (T - 23)$ with $T(0) = 33$. There's an unknown proportionality constant k but one can use the information in the problem to work out its value later. **(b)** Given a proposed exact solution to an initial value problem, $T(t)$ in order to verify it is indeed the exact solution, one has to (i) check that it satisfies the rate equation and (ii) check that it satisfies the initial condition. Both steps are necessary. There may be lots of functions which satisfy (i) or (ii) but if the IVP has an exact solution then there is only one function which satisfies both (i) and (ii) simultaneously. Many students did not show (i). Here it is: $T'(t) = \frac{d}{dt}[23 + 10e^{kt}] = 10ke^{kt}$ but this is supposed to also equal $k \cdot (T-23)$ according to the rate equation, but since $T(t) = 23+10e^{kt}$, then $T - 23 = 10e^{kt}$ and $k \cdot (T - 23) = 10ke^{kt}$ so this given function **does indeed** satisfy the rate equation. **(c)** To obtain an estimate of $T'(0)$ one uses the microscope approximation or recall from Lab how one can use extra given temperature data to estimate the rate of change of temperature. Thus $T'(0) \approx \frac{\Delta T}{\Delta t} = \frac{T(0) - T(75)}{0 - 75} = \frac{33 - 30}{-75} = -0.04$. Note this T' would have units of degrees per minute. You can also use a timescale of hours, in which case, $T'(0) \approx \frac{\Delta T}{\Delta t} = \frac{T(0) - T(1.25)}{0 - 1.25} = \frac{33 - 30}{-1.25} = -2.4$. **(d)** Since one now has an estimate for a rate of change of T, one can use the rate equation to estimate k. $T(t) = k \cdot (T(t) - 23)$ so $T'(0) = k \cdot (T(0) - 23) = k(33 - 23) = 10k$ and $T'(0) \approx -0.04$ so $k \approx -0.004$. Again, you could have used the timescale of hours instead of minutes so that $T'(0) = k \cdot (T(0) - 23) = k(33 - 23) = 10k$ implies $k \approx -0.24$ **(e)**. Clearly this is a pretty awful estimate for k since it depends on an estimate for $T'(0)$ which uses temperature values 75 minutes apart! This would therefore produce a very inaccurate estimate of the time of death. To improve an estimate for k one should measure the temperature of the body at closer time intervals, and more often.

#3 The "Been There, Done That" Problem. Ahh, the lovely S-I-R disease model. **(a)** This question is just asking how long it takes for the disease to progress (on average) through an infected person. The answer is 1/0.05=20 days since the constant b is the reciprocal of the recovery time. **(b)** This question is asking whether the disease is an epidemic right now or not–if there is an epidemic, the disease must be spreading at time $t = 0$. You can determine this in a number of ways: one is to find $I'(0)$. If $I'(0) > 0$ then the disease is spreading. Another way to address this problem is to find the threshold value $S_* = b/a$ for this model and evaluate whether $S(0) > b/a$. If $S(0) > S_*$ then there is an epidemic. Of course you need to show your calculations for how you compute the numbers which inform your evaluation of the state of the disease. It turns out that $S_* = 0.05/0.004 = 12.5 << 1000 = S(0)$; from the rate equation and initial conditions one can obtain $I'(0) = 0.004 * S(0) * I(0) - 0.05 * I(0) = 120 - 1.5 = 118.5 > 0$. In either case we know that the disease is spreading (and you only need one of these to indicate the answer). **(c)**. Since the disease is spreading, there is an epidemic, and one knows $S(0) > S_*$. **(d)** One knows $I'(0) = 118.5$ and $I(0) = 30$ so $I(t) \approx I(0) + I'(0)t$ and the equation of the tangent line is $y = 30 + 118.5t$.

#4 The Visual Problem. Eeek, slope fields and Euler's Method. The idea here is to combine graphical understanding with computational understanding. **(a)** All the graphs must start with the same slope of -2 and then have shorter "branches off." They should generally follow the "flow" of the slope field, with the idea that the successive approximations are tending towards some smooth parabolic looking curve. **(b)** Number crunching Euler's Method. One should obtain $y(2) \approx -2$ with $\Delta x = 2$, $y(2) \approx 1$ with $\Delta x = 1$ and $y(2) \approx 2.5$ with $\Delta x = 0.5$. **(c)** BONUS. Bonus points are extremely hard to get so your answers have to be exceedingly precise. The exact value of $y(2)$ we know is greater than all of the approximate values computed in (b) because we know that Euler's Method produces more accurate approximations as step size decreases and we notice that the approximate values increased in size (from -2, to 1 to 2.5). We also notice that the successive approximations appear to indicate the exact solution is concave up. When one uses a tangent line approximation to a curve which is concave up your estimate will be an under-estimate. Thus the estimates in (b) are also under-estimates of the exact value of $y(2)$. One can also compute the exact solution to $y' = 3x - 2, y(0) = 2$ since we might recall that the function whose derivative is $3x - 2$ is $3\frac{x^2}{2} - 2x + C$ and thus the function which solves the IVP exactly is $y(x) = 3\frac{x^2}{2} - 2x + 2$ and thus $y(2) = 4$ which is greater than all the approximated calues in (b).