Math 114: Calculus I. **Midterm 3** Name: _____

Instructor: Ramin Naimi Fri, 19 Nov 2004

Closed book. Closed notes. NO CALCULATORS. 20 points per problem. Please write very legibly. Circle all scratch work and write "Do not grade" on it.

Do **only five** of the following problems; write here ____ the problem you choose **not** to do.

1. (a) Find the linear approximation of $f(x) = e^x$ at $x = 0$. Show all work.
   (b) Find the quadratic approximation of $f(x) = e^x$ at $x = 0$. Show all work.

2. Find each of the following limits. (Some do and some do not need L'Hopital's Rule.) Show all work.
   (a) $\lim_{x\to 0} \dfrac{e^{2x} - 1 - 2x}{x^2}$
   (b) $\lim_{x\to \infty} \dfrac{\ln x}{e^{1-x}}$
   (c) $\lim_{x\to \infty} (1 + \dfrac{2}{x})^{3x}$

3. Find all local and absolute extrema of $f(x) = x^3 - 9x$ on the interval $[-2, 1]$. Explain all work.

4. (a) Given any differentiable function $f(x)$, write the equation of its tangent line at $x = a$ in terms of $f(a)$ and $f'(a)$. Explain *briefly* how you obtain this equation.
   (b) Use part (a) above to obtain the recursion formula for Newton's Method: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$. Explain your work, and draw a picture.

5. Let $f(x) = x^5 + 2x - 1$.
   (a) Does f have an inverse function? Why?
   (b) Find the derivative of f^{-1} at -1. Explain all work.

6. A camera tracks the launch of a vertically ascending spacecraft. The camera is located at ground level 2 miles from the launch pad. When the spacecraft is 3 miles up, the camera angle (measured from the horizontal) is changing at a rate of $1/2$ radians per second. Find the speed of the spacecraft and give its units. Show all work.