Math 110 Fall 2007 Quiz Ten

SHOW ALL YOUR WORK AND EXPLAIN ALL YOUR ANSWERS

1. Find the two positive numbers x and y which satisfy the given requirements:

   - ☐ The product of the two numbers is **192**.
   - ☐ The sum of the two numbers is a **minimum**.

   a. (*1 point*) Write down an equation relating x and y using the information above.

   $$xy = 192$$

   b. (*2 points*) Write an equation for S, the sum of the two numbers, that only has ONE VARIABLE (x or y) in it.

   $$S = x + y = x + \frac{192}{x}, \quad 0 < x < \infty$$

   c. (*4 points*) Find the values of x and y which minimize S

   $$S' = 1 - \frac{192}{x^2} = 0$$

   $$1 = \frac{192}{x^2} \Rightarrow x^2 = 192$$

   $$x = \sqrt{192} = \sqrt{4 \cdot 48} = \sqrt{4 \cdot 16 \cdot 3} = \sqrt{4}\sqrt{16}\sqrt{3} = 2 \cdot 4 \cdot \sqrt{3} = 8\sqrt{3}$$

   S': $-$ ↓ $\sqrt{192}$ ↑ $+$

   $$S'' = 2 \cdot \frac{192}{x^3} = 2 \cdot \frac{192}{(8\sqrt{3})^3} > 0$$

   So $x = 8\sqrt{3}$ is a local minimum

   d. (*2 points*) Check that the values of x and y which minimze S actually produce the smallest value S can be.

   $$y\sqrt{192} = 192$$

   $$y = \frac{192}{\sqrt{192}} = \sqrt{192}$$

   $S''(\sqrt{192}) > 0$
   So $\sqrt{192}$ is location of local MIN by 2nd Deriv Test

   e. (*1 point*) What is the minimum value of the sum S for two numbers whose product is 192?

   $$S = \sqrt{192} + \sqrt{192} = 2\sqrt{192} = 2(8\sqrt{3}) = 16\sqrt{3}$$