Math 110: Basic Calculus 1
Fall 2007

Quiz 9 : Extrema and Curve Sketching
Prof. R. Buckmire

Lab Time: **Your Name:** BUCKMIRE

GOAL: This quiz is designed to illuminate your understanding of how to analyze the graphical behavior of functions in terms of extrema, concavity and derivatives..

1. (*12 points*) **Multiple Choice.** Indicate your answer to the following multiple choice questions (1 point) by selecting the appropriate box. Your explanation of your answer is worth 2 points.

(a). Which of the following statements is always true?

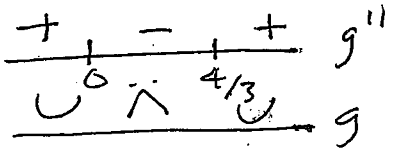
(A) ☐ All local extrema are also global extrema.
(B) ☑ All global extrema are also local extrema.
(C) ☐ Some global extrema are local extrema.
(D) ☐ No local extrema are global extrema.
(E) ☐ None of the above statements is true.

A global extremum is chosen from a local extremum so every global extrema is a local extrema

(b). Consider an unknown function $g(x)$ where $g'(x) = x^2(x-2)$. It has

(A) ☐ no inflection points.
(B) ☐ one inflection point.
(C) ☑ two inflection points.
(D) ☐ three inflection points.
(E) ☐ an unknowable number of inflection points.

$$g'' = 2x(x-2) + x^2 \cdot 1$$
$$= 2x^2 + x^2 - 4x$$
$$= 3x^2 - 4x = x(3x-4)$$



(c). Consider an unknown function $g(x)$ where $g'(x) = x^2(x-2)$. It has

(A) ☐ no critical points.
(B) ☐ one critical point.
(C) ☑ two critical points.
(D) ☐ three critical points.
(E) ☐ an unknowable number of ~~inflection~~ critical points.

$0 = x^2(x-2) \Rightarrow x = 0$ and $x = 2$

2 critical points

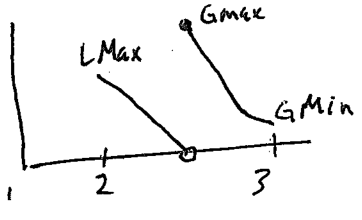
(d). Consider an unknown function $M(x)$ where all you know is that $M(x)$ is decreasing at every point in the interval [2, 3]. Which of the following must be true?

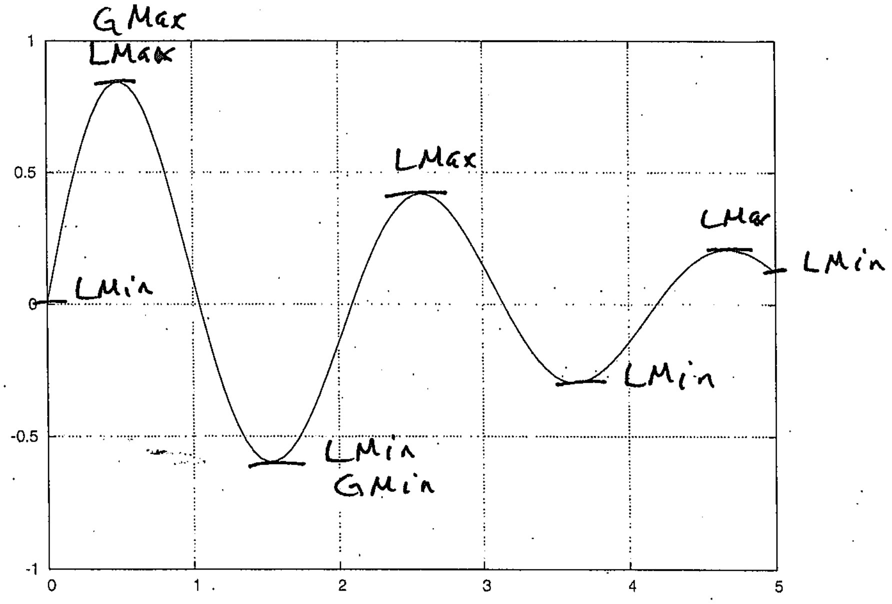
(A) ☐ $M(x)$ has a local minimum at $x = 2$.
(B) ☐ $M(x)$ has a global minimum at $x = 2$.
(C) ☑ $M(x)$ has a local maximum at $x = 2$.
(D) ☐ $M(x)$ has a global maximum at $x = 2$.
(E) ☐ More than one of the above statements must be true.

Continuity is <u>NOT</u> given so extreme value theorem does not apply

$M(x)$ could look like

This is the difference between being told $g' < 0$ on interval versus





2. *(8 points)* Consider the graph of the function $f(x) = e^{-0.2x}\sin(x)$ on $[0,5]$. Label **all** local maxima with LMax; similarly, label **all** local minima with **LMin**. Then, label **all** global maxima with GMax; similarly, label **all** global minima with GMin.

BONUS *(5 points)* Consider the function $F(x) = x^2e^{-x}$. Sketch a graph of the function $F(x)$ on its domain after clearly identifying the locations of all extrema and inflection points.

$F'(x) = 2xe^{-x} + x^2(-e^{-x}) = e^{-x}(2x - x^2)$

Critical Points occur $F' = 0$ or F' DNE

$2x - x^2 = 0$

$(2-x)x = 0$

$x = 0$ or 2

$-$ $+$ $-$ F'

$*$ 0 $*$ 2

↓ ↑ ↓

~~0 is~~ LMin is at 0

LMax is at 2

$F''(x) = \cancel{2}-e^{-x}(2x - x^2) + e^{-x}(2-2x)$

$= \cancel{e^{-x}(1\ldots)}\, e^{-x}[x^2 - 2x - 2x + 2]$

$= e^{-x}[x^2 - 4x + 2]$

$x = \frac{4 \pm \sqrt{4^2 - 4(2)(1)}}{2}$

$= \frac{4 \pm \sqrt{8}}{2} = 4 \pm 2\sqrt{2}$

$= 2 \pm \sqrt{2}$

~~$+$ $4-2\sqrt{2}$ $+$ $4+2\sqrt{2}$ $+$~~ F''

0

$+$ $-$ $+$ F''

∪ $2-\sqrt{2}$ ∩ $2+\sqrt{2}$ ∪

inflexion points