SHOW ALL YOUR WORK AND EXPLAIN ALL YOUR ANSWERS

a. *(5 points.)* Evaluate $\lim_{x\to\infty} \frac{(\ln(x))^3}{x}$. Explain your answer.

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{3(\ln(x))^2 \cdot \frac{1}{x}}{1} = \lim_{x\to\infty} \frac{3(\ln x)^2}{x} \overset{L'H}{=} \lim_{x\to\infty} \frac{3 \cdot 2 \ln(x) \cdot \frac{1}{x}}{1}$$

$$= \lim_{x\to\infty} 3 \cdot 2 \cdot \frac{\ln(x)}{x} \overset{L'H}{=} \lim_{x\to\infty} \frac{3 \cdot 2 \cdot 1 \cdot \frac{1}{x}}{1} = 0$$

b. *(5 points.)* Evaluate $\lim_{x\to\infty} \frac{(\ln(x))^{1000000}}{x}$. Explain your answer.

Use L'Hopital's Rule 1,000,000 times and you'll end up with $\lim_{x\to\infty} \frac{1000000!}{x} = 0$

$x \to \infty$ faster than $(\ln x)^{1000000} \to \infty$

so $\lim_{x\to\infty} \frac{(\ln(x))^{1000000}}{x} = 0$

c. **BONUS** *(5 points.)* Evaluate $\lim_{x\to\infty} \frac{(\ln(x))^m}{x}$ where m is **any** real number. Explain how (or if) the value of the limit depends on the values of m.

When $m=0$ $\lim_{x\to\infty} \frac{1}{x} = 0$

When $m<0$

$\lim_{x\to\infty} \frac{(\ln x)^m}{x} = \lim_{x\to\infty} \frac{1}{x(\ln x)^{-m}} = 0$

$$\lim_{x\to\infty} \frac{(\ln x)^m}{x} = \begin{cases} 0, & m=0 \\ 0, & m>0 \\ 0, & m<0 \end{cases}$$

The value of the limit is zero for all values of m.