Lab Time: **Your Name:**

GOAL: This quiz is designed to illuminate your understanding of the Chain Rule and to give you practice in the verbal communication of mathematical concepts and provide an oppoprtunity to demonstrate critical thinking.

Three students, Madison, Sydney and Tyler are discussing a particularly evil problem their hard-working Calculus professor has assigned them:

$$\text{Given } f(x) = (x^2 - 5x + 4)^{1000}, \text{ find } f'(x).$$

Tyler: Ohhhh, *censored*! Is s/he kidding, this looks like one of those functions that doesn't have a derivative for any value of x!

Madison: I don't know, it doesn't look *that* hard. I think the exponential function $x^2 - 5x + 4$ can be factored as $(x + 1)(x - 4)$ and doing that must help us figure out the derivative.

Sydney: Let's not panic, folks. The professor said when you're going to compute a derivative you need to be clear about what rule you're going to use to differentiate it. I think for this function we can use the Power Rule to find the answer.

Tyler: Oh, yeah, the Power Rule. That's the one where the derivative of x to any power n equals the function $(n - 1)x^n$.

Sydney: My bad, isn't this quiz supposed to be about the Chain Rule? I think we can use it to find the derivative, we just need to figure out which function is composed with which and in what order. I think the two functions are $p(x) = x^{1000}$ and $q(x) = x^2 - 5x + 4$ but I'm not sure which is the "inner" function and which is the "outer" function.

Madison: Hmmm, well I don't think we need the Chain Rule at all. After I've factored the polynomial $x^2 - 5x + 4$ I can easily just expand each linear term by raising them to the 1000th power and then differentiate the resulting polynomial term by term.

Tyler: I think Sydney's on to something. I don't think the order of composition of the functions matters, the derivative of $p(x)$ composed with $q(x)$ (or $q(x)$ composed with $p(x)$) is simply $p'(x)q'(x)$.

Write a short essay explaining the misunderstandings the three students have of the chain rule and differentiation in general. Discuss the comparative level of understanding of Calculus demonstrated by Madison, Sydney and Tyler. A good approach would be to carefully examine each of the mathematically relevant statements and then say whether and why each of the statements by the students is either true or false. You should consider finding the derivative in question and stating what rules you would use to find it. To obtain full credit, you will have to analyze at least five (5) mathematically relevant statements made by the students. **You must be careful not to make any incorrect statements yourself in your explanations. PROOFREAD YOUR ANSWER. You may want to use the back of the page to complete your response.**