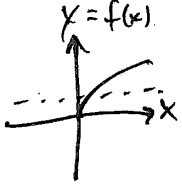
Math 110 Fall 2007 Quiz Two

SHOW ALL YOUR WORK AND EXPLAIN ALL YOUR ANSWERS

Consider the two functions $f(x) = \sqrt{x}, x \geq 0$ and $g(x) = x^4, -\infty < x < \infty$.

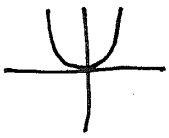
(a) *(3 points)* Is $f(x)$ invertible? If not, why not? If it is invertible, say why. Is $f(x)$ even, odd or neither? EXPLAIN YOUR ANSWERS.



Yes, $f(x) = \sqrt{x}, x \geq 0$ is invertible since it passes the horizontal line test.

No, $f(x) = \sqrt{x}$ is NOT EVEN OR ODD ($f(-x)$ is undefined when $x \geq 0$)

(b) *(3 points)* Is $g(x)$ invertible? If not, why not? If it is invertible, say why. Is $g(x)$ even, odd or neither? EXPLAIN YOUR ANSWERS.



No, $g(x)$ is not invertible since it is NOT one-to-one, it does not pass the horizontal line test!

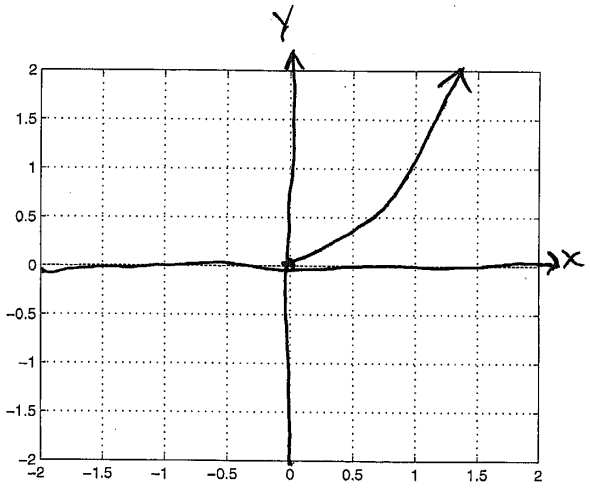
Yes, $g(x)$ is even. $g(-x) = (-x)^4 = (-1)^4 x^4 = x^4 = g(x)$.

(c) *(2 points)* Compute $(g \circ f)(x)$ and $(f \circ g)(x)$ and give their domains. Are these functions different **(NOTE: two functions are the same only if both FORMULAS AND both DOMAINS are identical!)**

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^4 = x^2, \quad x \geq 0$

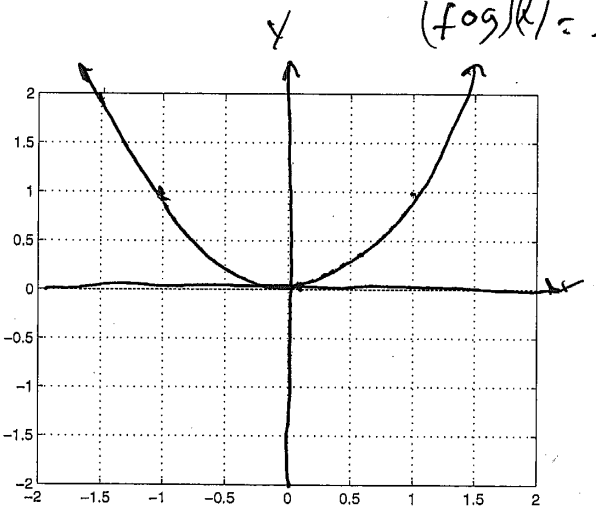
$(f \circ g)(x) = f(g(x)) = f(x^4) = \sqrt{x^4} = x^2, \quad -\infty \leq x \leq \infty$

(d) *(2 points)* Sketch the graphs of $(g \circ f)(x)$ and $(f \circ g)(x)$ on the axes below.



Graph of $g \circ f$

INVERTIBLE!



$(f \circ g)(x) = x^2, -\infty < x < \infty$

Graph of $f \circ g$

NOT INVERTIBLE!