Math 110 Fall 2007 **BONUS** Quiz 3

SHOW ALL YOUR WORK AND EXPLAIN EVERY ANSWER

1. (*5 points*). The speed S of blood that is r centimeters from the center of an artery is given by the equation

$$S = C(R^2 - r^2)$$

where C is a constant, R is the radius of the artery, and S is measured in centeimeters per second. Suppose a drug is administered and the artery begins to dilate at a rate of $\frac{dR}{dt}$. At a constant distance r, find the rate at which S changes with respect to t for $C = 1.76 \times 10^5$, $r = 1.2 \times 10^{-2}$ and $\frac{dR}{dt} = 10^{-5}$.

$$\frac{dS}{dt} = \frac{dS}{dR}\frac{dR}{dt}$$

$$= 2CR\frac{dR}{dt}$$

$$= 2 \cdot (1.76 \times 10^5)(R)(10^{-5})$$

$$= 3.52R \text{ cm/s}$$

2. (*5 points*). Suppose we define two new functions, called hyperbolic trigonometric functions, where hyperbolic cosine is denoted $\cosh(x)$ and hyperbolic sine is denoted $\sinh(x)$. Given that

$$\frac{d}{dx}\sinh(x) = \cosh(x) \text{ and } \frac{d}{dx}\cosh(x) = \sinh(x) \text{ and } \cosh(0) = 1 \text{ and } \sinh(0) = 0$$

Show that $\cosh^2(x) - \sinh^2(x) = 1$ for every value of x.

HINT: show that $\frac{d}{dx}[\cosh^2(x) - \sinh^2(x)] = 0$ **and explain why that means** $\cosh^2(x) - \sinh^2(x) = 1$

$$\frac{d}{dx}[\cosh^2(x) - \sinh^2 x] = 2\sinh x \cosh(x) - 2\sinh x \cosh x$$

$$= 0$$

This means that

$$\cosh^2 x - \sinh^2 x = \text{constant}$$

At $x = 0$

$$\cosh^2 0 - \sinh^2 0 = \text{constant}$$

$$1^2 - 0^2 = \text{constant}$$

$$1 = \text{constant}$$

$$\boxed{\cosh^2 x - \sinh^2 x = 1}$$