Math 110 Fall 2007 **BONUS Quiz 2**

SHOW ALL YOUR WORK

Consider the functions $f(x) = |x|$ and $g(x) = x$. Let $P(x) = f(x)g(x)$ and $Q(x) = \frac{f(x)}{g(x)}$.

a. *(3 points.)* Compute $P'(1)$ and $Q'(1)$ or explain why these derivatives do not exist. (HINT: Can you use the Product and Quotient Rules or do you have to compute the derivatives algebraically?)

$P(x) = |x|\, x$

$P'(x) = (|x|)' \cdot x + |x| \cdot (x)'$

when $x > 0$, $(|x|)' = 1$

$x < 0$, $(|x|)' = -1$

$x = 0$, $(|x|)'$ DNE

$P'(1) = 1 \cdot 1 + 1 \cdot 1 = \boxed{2 = P'(1)}$

$Q(x) = \frac{|x|}{x}$

$Q'(x) = \frac{(|x|)' \cdot x - |x| \cdot x'}{(x)^2}$

$Q'(1) = \frac{1 \cdot 1 - 1 \cdot 1}{1^2} = 0$

$\boxed{Q'(1) = 0}$

b. *(3 points.)* Compute $P'(0)$ and $Q'(0)$ or explain why these derivatives do not exist. (HINT: Can you use the Product and Quotient Rules or do you have to compute the derivatives algebraically?)

At $x = 0$, $|x|'$ DNE, so we can not use Product Rule OR Quotient Rule

Use Algebraic Def^n

$P'(0) = \lim_{h \to 0} \frac{P(0+h) - P(0)}{h}$

$= \lim_{h \to 0} \frac{|h|\, h - |0| \cdot 0}{h}$

$= \lim_{h \to 0} \frac{|h| \cdot \cancel{h}}{\cancel{h}} = \lim_{h \to 0} |h| = 0$

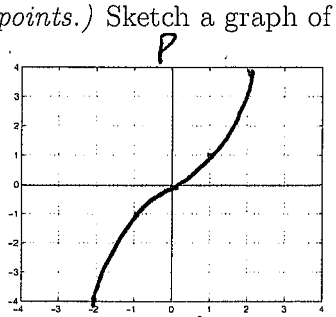
$Q'(0) = \lim_{h \to 0} \frac{Q(0+h) - Q(0)}{h}$

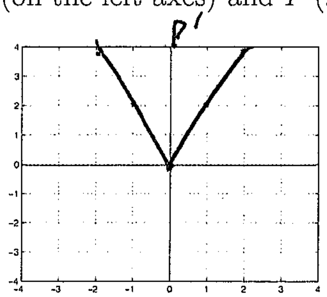
$= \lim_{h \to 0} \frac{\frac{|h|}{h} - \frac{|0|}{0}}{h}$ $Q(0)$ is not defined

$Q'(0)$ does not exist because $Q(0)$ is undefined.

c. *(2 points.)* Sketch a graph of $P(x)$ (on the left axes) and $P'(x)$ (on the right axes).

$P(x) = \begin{cases} x^2, & x > 0 \\ -x^2, & x < 0 \end{cases}$

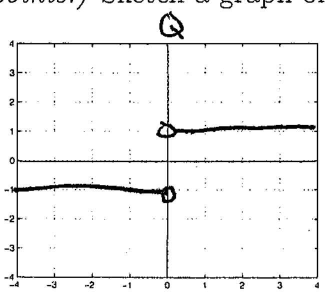


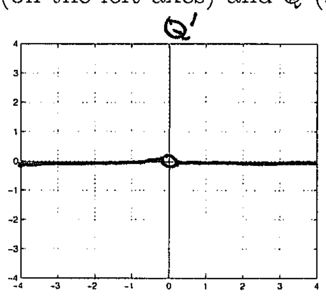


$P'(x) = \begin{cases} 2x, & x > 0 \\ -2x, & x < 0 \end{cases}$

d. *(2 points.)* Sketch a graph of $Q(x)$ (on the left axes) and $Q'(x)$ (on the right axes).

$Q(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$





$Q'(x) = \begin{cases} 0, & x > 0 \\ 0, & x < 0 \end{cases}$