Application Of Derivatives: Optimization

Optimization of a function refers to finding its *global* maximum or minimum value as well as the point(s) where this occurs. Recall:

A function f has a **global maximum** at x_0 in its domain if $f(x_0) \geq f(x)$ for *all* x in its domain.

To find a global maximum for a function on a *closed* interval, i.e. one which includes its endpoints, first find all the local maxima for the function on that interval. Evaluate the function at each of these points. The global maximum will be the largest of these function values.

To find a global maximum for a function on an *open* interval or the real line (i.e. where endpoints are not included), first find all the local maxima for the function on that interval. Evaluate the function at each of these points. IF a global maximum exists, it will be the largest of these function values. However, it is possible that the function may assume even larger values as its argument approaches one of the endpoints of the interval, so this must be checked. (In this case, there would be no global maximum.)

Write below the corresponding criterion for a global MINIMUM, as well as directions for finding it on a closed or open interval.

Optimization problems arise in mathematics itself and in its applications. In this class we will look at examples of optimization problems where our knowledge of first and second derivative tests will be helpful.

Examples

[1]. A woman pulls a sled which, together with its load, has a mass of m kg. If her arm makes an angle of θ with her body (assumed to be vertical) and the coefficient of friction (a positive constant) is μ, the least force, F, she must exert to move the sled is given by

$$F = \frac{mg\mu}{\sin\theta + \mu\cos\theta}.$$

If $\mu = 0.15$, find the global maximum and minimum values for F for $0 \le \theta \le \pi/2$. Give your answers in multiples of mg, the weight of the loaded sled.

[2]. An electric current, I, in amps, is given by

$$I = \cos(wt) + \sqrt{3}\sin(wt),$$

where $w \neq 0$ is a constant. What are the global maximum and minimum values of I?

[3]. Show that $x > 2\ln x$ for all $x > 0$. (*Hint: Find the minimum of* $f(x) = x - 2x$.) *Then use this result to show that* $e^x > x^2$ *for all positive* x.

[4]. Is $x > 3\ln x$ for all positive x?

[5]. Show that the rectangle of fixed perimeter P whose area is a maximum is a square.

[6]. A roman window is shaped like a rectangle surrounded by a semi-circle. If the perimeter is L feet, what are the dimensions of the window of maximum area?