Math 110: Some Practice Exam Problems

1. Use the rules of differentiation to find the first derivative for the following functions. **DO NOT SIMPLIFY YOUR ANSWER.**

   (a) $f(t) = t^{7/2} - 3t + 5$

   (b) $y = \dfrac{e^x}{x}$

   (c) $g(x) = \sqrt[3]{(1-x^3)^2}$

   (d) $z = 2^x \ln x$

   (e) $h(x) = \dfrac{\ln(\sin(2x))}{\tan(x)}$

2. Use the **limit definition** of the derivative to find the derivative of the function $f(x) = \dfrac{1}{x-2}$. **Show all of your work!**

3. Find the equation for the line tangent line to the graph of $q(x) = \dfrac{3^x}{x}$ at the point $(1, 3)$.

4. What is the microscope approximation for $y = \sin(x)$ at the point $\left(\dfrac{\pi}{12}, \dfrac{\sqrt{6}-\sqrt{2}}{4}\right)$?

   Use this microscope approximation to estimate $\sin\left(\dfrac{1}{4}\right)$.

5. A textbook has been dropped from a cliff! Using the table of values given below, make an estimate of the instantaneous velocity of the textbook at time $t = 2$ seconds. Justify your answer in writing.

Time	Height
t	h(t)
2.0000	936.00000000
2.0001	935.99359984
2.0010	935.93598400
2.0100	935.35840000
2.1000	929.44000000
3.0000	856.00000000

6. Consider the equation $y^2 + xy - x^2 = 5$. Find $\dfrac{dy}{dx}$ at the point $(1, 2)$ and also at the point $(1, -3)$.

7. Below is a simple polynomial function and its first two derivatives (in factored form)

$$\begin{aligned} f(x) &= x^4 - 4x^3 + 16x \\ f'(x) &= 4(x+1)(x-2)^2 \\ f''(x) &= 12x(x-2) \end{aligned}$$

(a) Determine on what intervals the function $f(x)$ is increasing.

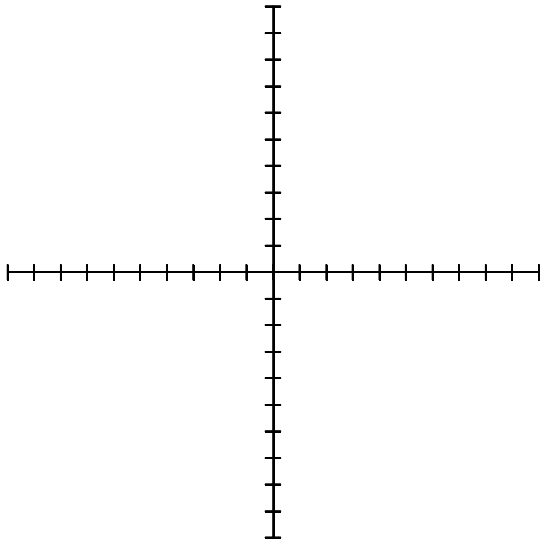
(b) Determine where the function $f(x)$ has relative extrema.

(c) Determine on what intervals the function $f(x)$ is concave down.

(d) Determine where the function $f(x)$ has points of inflection.

(e) One of the x-intercepts of the function $f(x)$ is $x = 0$. The other one lies between -2 and -1. Use Newton's Method with an initial guess of $x_1 = -2$ to find the other intercept to two decimal places.

(f) Graph the function $f(x)$ below, labeling your scale and all important points (intercepts, extrema, points of inflection, etc.).



8. Consider the Initial Value Problem (IVP)

$$\begin{aligned} C' &= -2C \\ C(0) &= 1. \end{aligned}$$

(a) Use Euler's method with a time step of $\Delta t = \dfrac{1}{10}$ to fill in the empty boxes in the table below and **find an estimate of** $C\left(\dfrac{1}{3}\right)$**.**

t	C	C'	ΔC
0	1	XXXXXXXX XXXXXXXX XXXXXXXX	XXXXXXXX XXXXXXXX XXXXXXXX
0.1	$\frac{4}{5}$		
0.2			
0.3			
0.4	$\frac{256}{625}$	XXXXXXXX XXXXXXXX XXXXXXXX	XXXXXXXX XXXXXXXX XXXXXXXX

(b) Verify that $c(t) = e^{-2t}$ is the solution to the IVP above. Show all of your work. Then use your calculator to compute $c\left(\dfrac{1}{3}\right)$.

(c) Graph the piecewise linear approximate solution you found using Euler's Method as well as the actual solution $c(t)$ on the same plot, for $0 \leq t \leq 0.4$.

9. Determine if the function $h(x) = \dfrac{3x^2 + 1}{2x^2 - x}$ has any vertical or horizontal asymptotes by evaluating the appropriate limits. If it does have vertical or horizontal asymptotes, give the equations for those asymptotes.

10. On the interval $[-1, 2]$, find the x-values where the global maximum and minimum occur for the function $f(x) = x^4 - 4x^3 + 3x^2$.

11. Suppose $y' = y \ln(y)$ for $y > 0$. Find a formula for y'' in terms of y. Find equilibrium value(s) and possible inflection value(s). Make a table to indicate how the slope and concavity of y behave for different values of y, then use this table to sketch approximate solutions to the rate equation.

12. Of all rectangles with area A, which has the shortest diagonals?