Limits and Continuity

Although you do not need to know the formal definition of a limit, as developed in lab, you do need to know that $\lim_{x\to a} f(x)$ exists if and only if $\lim_{x\to a-} f(x)$ and $\lim_{x\to a+} f(x)$ exist and are equal. Then $\lim_{x\to a} f(x)$ equals this common value.

You do need to be familiar with the rules for limits and the particular limits on the *Limits* handout. This means you need to be able to apply them yourself and to recognize when they are being used in a derivation.

You need to know what it means, both graphically and in terms of a limit, to say that a function f is continuous at $x = a$.

1. Given $f(x) = \dfrac{x^2 - 9}{x - 3}$
   (a) Evaluate $\lim\limits_{x\to 3^+} f(x)$
   (b) Evaluate $\lim\limits_{x\to 3^-} f(x)$
   (c) Evaluate $\lim\limits_{x\to 3} f(x)$
   (d) Is $f(x)$ continuous at $x = 3$?

2. Use the fact that $\ln(x)$ is continuous for $x > 0$ to evaluate $\lim\limits_{y\to 0} \dfrac{\ln(1+y)}{y}$.

Definition of the Derivative and Local Linearity

You should know and be able to apply both forms of the "limit" definition of the derivative.

You should know that the derivative is interpreted both as the "instantaneous rate of change" of a function at a point and as the slope of the line tangent to the graph of the function at that point. You should be able to draw a diagram illustrating the relationship between a function graph and a tangent line, and to correctly use the symbols Δy, Δx, dy and dx in labeling this diagram.

You should be able to find the equation of the line tangent to the graph of a function at a point where it is differentiable, and to use this approximate the function near that point. You should know to what the terms *local linearization*, *Microscope Approximation* and *first-degree Taylor polynomial* refer. You should know Taylor's Theorem and, in particular, know the limit properties of the error term.

You should know that every differentiable function is continuous, but not every continuous function is differentiable.

3. (a) What is the difference between saying that a functon is continuous at a point and saying that the function is differentiable at that point?

   (b) Give an example of a function that is continuous at $x = 1$ but is not differentiable there.

4. Let $f(x) = \dfrac{1}{x-1}$. Find $f'(x)$ using the definition of the derivative. Is f differentiable at $x = 1$?

5. Find the equation for the line tangent to the curve $y = x^2$ at the point $(-3, 9)$.

6. Consider the function $g(x) = x^2 + 4$. There are two points on the graph of this function that have tangent lines which pass through the origin $(0, 0)$. Find those two points.

Rules for Differentiation

You need to know the general rules for differentiation and the derivatives of the particular elementary functions listed on the *Derivatives* handout. You need to be able to apply them yourself and to recognize when they are being used in a derivation.

You should be familiar with both the "prime" and "differential" notations to indicate a derivative. You should know how to differentiate an equation implicitly defining one variable as a function of another, and to solve for the derivative of the dependent variable with respect to the independent variable.

7. Differentiate the following functions. Do NOT simplify.

   a. $p(x) = -4x^{\frac{1}{3}}$
   b. $q(u) = 5\tan(u)$
   c. $r(\theta) = \sin(\theta)e^{\theta}$
   d. $h(t) = \dfrac{t^4}{\ln(t)}$
   e. $g(x) = \cos(x + e^{5x})$

8. The function $s(x) = q(p(x))$ is a composite function built from the functions in Problem #7 above. Use the Chain Rule to find the derivative of the function $s(x)$. Do NOT simplify.

9. Suppose we have functions $f(x)$ and $g(x)$ such that $f(1) = 4$, $f'(1) = -2$, $g(1) = 9$, and $g'(1) = 3$. Let $h(x) = f(x)/g(x)$. Find $h'(1)$.

10. Suppose $x^2y + xy^3 = 1$. Find $y'(x)$.