Math 110 Fall 2000

Report on Exam 1

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Point Distribution (N = 70)

Range	102+	99+	96+	91+	88+	85+	80+	77+	74+	69+	66+	65-
Grade	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Frequency	15	9	5	11	8	5	5	2	3	2	1	4

<u>Comments</u>

Overall

#1 Proportionality and Linear Functions. The introduction of Devon, Chris and Lee. As it turned out, Devon is completely clueless, Lee is mostly correct, and Chris gets everything right! The main mistake people made here was on the first two statements. If A is proportional to B that means that $A = kB$ where k is non-zero constant. That's it! Separately, it is also true that the definition of a linear function is that it's change in output is proportional to its change in input, so $\Delta A = k \Delta B$. One needs to be careful to note the direction of the implication in Chris's and Lee's statements. Take Lee's statement: IF a function is linear, THEN change in output is proportional to change in input. This statement is true in both directions. If change in output is proportional to change in input, then the function is linear. However, Chris' statement is only true in the direction it was written: IF A is proportional to B, THEN $\Delta A = k\Delta B$. A number of people realized the statement is untrue if reversed and thus incorrectly declared Lee's statement false. The rest of the question is about notation, ensuring that everyone understand the meaning of ΔA (change in quantity A) and A' (rate of change of A, which we now know is also the DERIVATIVE of A).

#2 Euler's Method and IVPs. It's pretty clear that almost the entire class understands how to apply Euler's Method to approximate the solution to an IVP by filling out a table, and can explain the process. There were some slight scaling errors in plotting the piecewise linear functions, but nothing major. You just have to remember to draw straight lines between the data points computed from your table. The most difficulty in this problem seemed to come from approximating $C(3/4)$. Many people found an approximation by using Euler's Method again with $\Delta t = 1/4$, which is a correct solution, but more work than we intended. If you look at your graph, you know that $C(3/4)$ must lie on the line between $(1/2, 1)$ and $(1, 3/4)$. So all you need to do is find the equation of this line and plug in the t value. The equation of the line ends up being $C(t) = -0.5t + 1.25$ and so $C(3/4) \approx 7/8$ using this method. If you used Euler's Method with $\Delta t = 1/4$ you got a more accurate answer. You also should have been able to check whether your answer made any kind of sense by seeing if it fell between $3/4$ and 1, as it must, from simply looking at your graph in the previous part.

#3 Slope Fields and IVPs. Most people were able to correctly identify the rate equation that went with the slope field. This was done either by noting that slopes were constant across rows, and hence depended only upon P, or by checking points. Some people confused two or more of the following: the rate equations (which equate P' to expressions involving P and possibly t), the right-hand sides of the rate equations (expressions involving P and possibly t), and solutions of a rate equation (which are unknown functions expressing the dependence of P on t). When it came to sketching graphs of the solutions passing through specific points, some people weren't sure what to do while others sketched plausible solutions by ignored the specific points in the instructions. All you need to do is follow the slopes "like a boat drifting in the current." A subtle point: many people who otherwise sketched correct graphs showed the lower graph joining or crossing the constant one. A moment's reflection will show you why this couldn't happen – at the point where the graphs joined there would need to be two slopes, not one!

#4 Modeling. In the first part of this problem, you needed to clearly state that an epidemic occurred only when $I' > 0$, then use the rate equation for I' to derive the condition $S \le b/a$ guaranteeing that no epidemic will occur. Some people just tried to argue from the meaning of b and a; this received partial credit, but is not adequate. (For one thing, it fails to account for the relationship of S to b/a.) Nearly everyone drew a correct compartment diagram, although many people introduced a new compartment, V, for those in the population who were vaccinated. As the instructions said to do, we simply lumped this together with R and gave full credit. Most people did well constructing the model, though a few still had the problems which showed up on the last quiz. Remember: every arrow out of a compartment corresponds to a term subtracted from the rate for that compartment, while every arrow into a compartment corresponds to a term added to the rate for that compartment. On the last question most people observed that neither the recovery nor the transmission coefficients were changed by the introduction of vaccination. This was true (and received some credit) but missed the main point. The threshhold value was derived starting with the condition $I' \le 0$ and using the rate equation for I'. It is only because this rate equation was unchanged by the introduction of vaccination that there was a threshhold value for S and that this value was still b/a. Had the I' equation been changed, it is possible that there would not have been any threshhold value for S or that, if one existed, it would have been different from b/a.