Name _____

Triangular Numbers

The numbers 1, 3, 6,... are called the first three *triangular numbers* since they may be represented by triangular patterns of dots as below:

- Draw pictures for a few more triangular numbers on the back.

- Make a table of the first 10 triangular numbers and, for any n, give a formula for the n-th triangular number. Call the nth triangular number T_n. To prove your formula, try to give a picture with dots which illustrates your result. (Notice that what you have actually found is a formula for the sum of the first n integers.)

- What is the sum of any two consecutive triangular numbers? That is find a formula for $T_n + T_{n+1}$. Prove your answer is correct, using algebra and your result from the result above. Now draw a picture with dots to illustrate this result.

- Prove that if T is a triangular number, then so is $9T + 1$.

- Explain why each number in this sequence is a triangular number:

$$1, 1 + 9, 1 + 9 + 81, ..., 1 + 9 + 9^2 + ... + 9^k, ...$$