Midterm #3. Math 460, Topology. Name: ______________________
Instructor: Ramin Naimi Mon 16 Apr 2001

Closed book. Closed Notes. Only the Definitions-Theorems handout allowed. 25 points per problem. Please write very legibly.

1. Use diagrams, with explanations wherever appropriate, to show that gluing two Möbius bands along their circle boundaries yields a Klein bottle: $M \cup_\partial M \simeq K$.

2. For each of the following topological spaces:

   (i) Determine (without proof) which are homeomorphic to each other.

   (ii) Determine which are manifolds. For each that is not a manifold, briefly explain why. For each that is a manifold, describe its boundary (if any), its dimension, and say (without proof) whether or not it is compact.

   (a) $S^1 \times [0,1]$ (b) $S^1 \times (0,1)$ (c) $S^1 \times \mathbb{R}$ (d) S^2 (e) $S^2 - \{(0,0,1),(0,0,-1)\}$

   (f) $[B_1(0,0) \cup B_1(5,0)]/\{(x,y) \sim (x+5,y) \text{ iff } x^2+y^2 \leq 1/4\}$

   (g) $[B_1(0,0) \cup B_1(5,0)]/\{(x,y) \sim (x+5,y) \text{ iff } x^2+y^2 < 1/4\}$

   (h) $[\overline{B_1(0,0)} \cup \overline{B_1(5,0)}]/\{(x,y) \sim (x+5,y) \text{ iff } x^2+y^2 = 1\}$

3. According to the classification of surfaces, each of the quotient spaces Q described below is homeomorphic to one of S^2, nT^2, or $n\mathbb{R}\mathrm{P}^2$, minus some number (possibly zero) of open disks. Determine exactly which of these is the case (and the number of removed disks, if any). Explain your reasoning.

   (a) Q is the quotient space determined by the identifications $AB \sim DC$, $BC \sim ED$, $HA \sim GF$, $GH \sim FE$ on an octagon $ABCDEFGH$.

   (b) Q is the quotient space determined by the identifications $BA \sim DC$, $BC \sim ED$, $HA \sim GF$, $GH \sim FE$ on an octagon $ABCDEFGH$.

4. Let $M = [0,1]^2/\{(0,y) \sim (1,1-y)\}$ denote the Möbius band, and let $q : [0,1]^2 \to M$ be the corresponding quotient map. Let $A = [0,1] \times \{1/2\} \subset [0,1]^2$. For $\epsilon = 1/4$, which familiar topological space is $\overline{N_\epsilon(q(A))}$ (the closure of the ϵ-nbhd of the core of M) homeomorphic to? Use diagrams and brief explanations to support your answer.