**Midterm #2.** Math 460, Topology.
Instructor: Ramin Naimi

Name: ____________________
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Closed book. Closed Notes. Only the Definitions-Theorems handout allowed, for problems 2-5. 20 points per problem. Please write very legibly.

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1. Give the definition of each of the following.
   (a) A **connected** topological space.
   (b) An **open cover** of a subset $A$ of a topological space $X$.
   (c) Given topological spaces $X$ and $Y$, the **product topology** on $X \times Y$.

2. In this problem, just find an appropriate map; you do not need to prove it's a homeomorphism.
   (a) Prove that any two closed intervals $[a, b]$ and $[c, d]$ are homeomorphic.
   (b) Let $D$ be the quotient space $\mathbb{R}/\{x \sim (x + 1)\}$. Which familiar topological space is $D$ homeomorphic to? Find a homeomorphism to support your answer. (Don't forget to show your map is well-defined.)

3. Prove the following theorem: The continuous image of a connected set is connected; i.e, if $f : X \to Y$ is a continuous map between topological spaces, and if $X$ is connected, then f(X) is connected.

4. Let $X_1$ and $X_2$ be topological spaces. For each $i = 1, 2$, we define the **projection map** $\pi_i : X_1 \times X_2 \to X_i$ by $\pi_i(x_1, x_2) = x_i$. Prove that every projection map is continuous.

5. Prove that a closed subset of a compact topological space is compact. (Hint: Let $A$ be a closed subset of $X$. If $F$ covers $A$, then $F \cup \{X - A\}$ covers $X$; why?)

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