Midterm #1. Math 460, Topology. Name: ____________________
Instructor: Ramin Naimi Wed 14 Feb 2001

Closed book. Closed Notes. 25 points per problem. Please write very legibly.

1. (a) (5 points) Give the definition of a **closed subset** of a metric space. Write a complete and grammatically correct sentence.

   (b) (20 points) Show by example that the union of an infinite collection of closed subsets of a metric space is not necessarily closed.

2. (a) (5 points) Give the definition of a **continuous map** from one metric space to another. Write a complete and grammatically correct sentence.

   (b) (20 points) Suppose (X_1, d_1) and (X_2, d_2) are metric spaces, and suppose $f : X_1 \to X_2$ is a function such that the preimage of every open set is open, i.e, for every open set $A_2 \subset X_2$, $f^{-1}(A_2)$ is open in X_1. Prove that f is continuous.

3. (a) (5 points) Give the definition of an **open subset** of a metric space. Write a complete and grammatically correct sentence.

   (b) (20 points) Let (X, d) be a metric space. Let $p_1, p_2, \cdots, p_n$ be arbitrary points in X. Prove that the set $X - \{p_1, p_2, \cdots, p_n\}$ is an open subset of X. Prove everything from definitions; i.e., you may not use any theorems or homework problems without providing proofs for them here.

4. (a) (20 points) Show that Figure (a) can be deformed into Figure (b) (or vice versa).

   (b) (5 points) Show that Figure (b) can be deformed into Figure (c) (or vice versa).