1. (a) If S^n is defined as the boundary of the closed unit ball in $\mathbb{R}^{n+1}$, then what is S^0?
   (b) The intersection of S^2 with the xy-plane in $\mathbb{R}^3$ is a circle. We call this a **great circle**, since no other circle on S^2 can have a larger radius. Similarly, the intersection of S^2 with the yz-plane in $\mathbb{R}^3$ is a great circle. What is the intersection of these two great circles?
   (c) Now one dimension higher. Denote points in $\mathbb{R}^4$ by (x, y, z, w). Prove *rigorously* that the intersection of S^3 with the xyz-hyperplane in $\mathbb{R}^4$ is S^2. We call such a 2-sphere a **great sphere**.
   (d) What is the intersection of two great spheres? (Take for example the great sphere cut out by the xyz-hyperplane and the great sphere cut out by the yzw-hyperplane). Prove your answer rigorously.

2. A torus T^2 can be defined as a **solid square** I^2 ($= [0,1] \times [0,1]$) with its opposite edges identified (with the "right" orientation). Thus, $T^2 = I^2/R$, where $R = \{(x,0) \sim (x,1)\ ,\ (0,y) \sim (1,y)\}$. Similarly, a 3-dimensional torus T^3 can be defined as a solid cube I^3 with its opposite faces identified (with the right orientation). Make this precise by giving an appropriate definition for R': $T^3 = I^3/R'$, where $R' = \cdots$.

3. T^2 can also be defined as $S^1 \times S^1$. We can informally explain how this definition is equivalent to the above definition ($T^2 = I^2/R$) by arguing as follows. For every $t \in I$, the two endpoints of $I \times \{t\} \subset I^2$ are identified into one point; so each $I \times \{t\}$ becomes homeomorphic to $S^1 \times \{t\}$. Therefore, I^2/R is homeomorphic to $S^1 \times I$ with $S^1 \times \{0\}$ identified with $S^1 \times \{1\}$ (with the "right" orientation). Thus, we get $S^1 \times S^1$. Give a similar informal argument to show $I^3/R' \simeq S^1 \times S^1 \times S^1$.

4. (a) What familiar space is a punctured 3-sphere (S^3 minus one point) homeomorphic to? Briefly explain why.
   (b) Let p be an arbitrary point in S^2. Then $\{p\} \times S^1$ is a scc in $S^2 \times S^1$. Draw a schematic picture of this. Call this scc C. Does C bound a disk in $S^2 \times S^1$?
   (c) What familiar space is $(S^2 \times S^1) - (N_\epsilon(C))^\circ$ (i.e., $(S^2 \times S^1)$ minus the interior of an ϵ-nbhd of C) homeomorphic to?

5. Explain why $S^3 \not\simeq S^2 \times S^1$ by stating a property (a topological invariant) that one has but the other doesn't.

Challenge Problems

6. (a) Explain how $S^2 \times S^1$ can be viewed as two solid tori glued together along their boundaries.
   (b) Explain how S^3 can be viewed as two solid tori glued together along their boundaries. Hint: view it as the boundary of $B^2 \times B^2$ ($\simeq B^4$).
   (c) Explain why the above does *not* imply $S^3 \simeq S^2 \times S^1$.