1. Draw two disjoint squares $ABCD$ and $A'B'C'D'$ on a piece of paper. Make sure to label the vertices in the order listed above. For example, in the first square, AB and AD are edges, while AC is a diagonal. For each of the identifications listed below, describe the resulting quotient space Q as follows:

   (i) Is Q a manifold? (If not, explain why. And do not describe Q any further — skip to the next Q.)
   (ii) Is Q compact? What is the boundary of Q, if any?
   (iii) Is Q orientable?
   (iv) According to the classification of surfaces, Q is homeomorphic to one of S^2, nT^2, or $n\mathbb{R}\mathrm{P}^2$, minus some number (possibly zero) of open disks. Determine exactly which of these is the case (and the number of removed disks, if any).

   (a) $AB \sim A'B'$, $CD \sim C'D'$. (Note: the orientations matter! For example, $AB \sim B'A'$ is not the same as $AB \sim A'B'$.)

   (b) $AB \sim A'B'$, $BC \sim B'C'$, $CD \sim C'D'$, $DA \sim D'A'$.

   (c) $AB \sim A'B'$, $CD \sim D'C'$.

   (d) $AB \sim A'B'$, $CD \sim AB$.

   (e) $AD \sim BC$, $A'D' \sim B'C'$, $AB \sim A'B'$, $CD \sim D'C'$.

   (f) $AB \sim A'B'$, $BC \sim B'C'$, $CD \sim C'D'$, $DA \sim A'D'$.

2. Draw an octagon $ABCDEFGH$ on a piece of paper. Let Q be the quotient space determined by the identifications $AB \sim DC$, $BC \sim ED$, $HA \sim GF$, $GH \sim FE$. Describe Q in terms of the same criteria as the previous problem.

3. Can the connected sum of two non-orientable surfaces be orientable? If yes, give an example. If not, explain your reasoning.

4. True or False: A surface is non-orientable iff it contains a Möbius band as a subset. Support your answer.

5. Let $C = \{(x, y, z) \in [0, 1]^3 \mid \text{at least two of } x, y, z \text{ are zero}\} \subset \mathbb{R}^3$. For $i = 0, 1, 2, 3$, let $h_i : C \to \mathbb{R}^3$ be given by: $h_0(x, y, z) = (x, y, z)$; $h_1(x, y, z) = (-x, y, z)$; $h_2(x, y, z) = (-x, -y, z)$; $h_3(x, y, z) = (-x, -y, -z)$.

   (a) Which of the maps h_1, h_2, h_3 are isotopic to h_0? Just answer without proof.
   (b) Give a definition of what it means for a 3-manifold to be orientable, by generalizing our definition of orientability for 2-manifolds.
   (c) Can you think of any nonorientable 3-manifolds?
   (d) Give a definition of what it means for an n-manifold to be orientable.