

1. Which of the following letters, when viewed as subsets of $\mathbb{R}^2$, are manifolds? Give a brief explanation for each one that is not a manifold.

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

2. How many non-homeomorphic 1-manifolds do you think there are? List as many as you can think of.
3. Is an open ball minus its center a manifold? More precisely, let $x \in \mathbb{R}^n$, $r > 0$. Is $B_r(x) - \{x\}$ a manifold? Prove your answer.
4. (a) Is $S^1/\{(1,0) \sim (-1,0)\}$ a manifold? Explain.
 (b) Is $S^1/\{(x,y) \sim (-x,-y)\}$ a manifold? Explain.
5. State which of the following spaces are homeomorphic to each other. You do not need to prove your answers; but give brief explanation or draw pictures (or both) to support them.
- (a) $X = \overline{B_1(0,0)} \subset \mathbb{R}^2$ (i.e., X is the closed unit disk in the plane).
 (b) $Y = X - B_{0.5}(0,0)$. (c) Y° (the interior of Y , where Y is viewed as a subspace of $\mathbb{R}^2$).
 (d) $X/\partial X$. (This means identify the whole boundary of X into one point – recall that $\partial X = S^1$.)
 (e) $S^1 \times [0,1]$. (f) $S^1 \times (0,1)$. (g) $S^1 \times [0,1)$. (h) $S^1 \times \mathbb{R}$. (i) $\mathbb{R}^2$
 (j) $S^2 = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$. (k) $S^2 - \{(1,0,0)\}$. (l) $S^2 - \{(1,0,0), (-1,0,0)\}$.
 (m) $S^2 - B_{0.5}(1,0,0)$.