1. (a) Let $X_1 = \mathbb{R}$, $\mathcal{T}_1 = \{(a, \infty) \mid a \in \mathbb{R}\} \cup \{\mathbb{R}, \phi\}$. Prove that $\mathcal{T}_1$ is a topology.
   (b) Let $(X_2, \mathcal{T}_2)$ be $\mathbb{R}$ with the standard topology (i.e., the topology induced by the Euclidean metric). Let $f : X_1 \to X_2$ and $g : X_2 \to X_1$ be given by $f(x) = x$ and $g(x) = x$. Is f continuous? Is g continuous? Prove your answers.

2. For $i = 1, 2$, let $(X_i, \mathcal{T}_i)$ be a topological space.
   (a) Show that if $\mathcal{T}_1$ is the discrete topology, then every function $f : X_1 \to X_2$ is continuous.
   (b) Show that if $\mathcal{T}_2$ is the indiscrete topology, then every function $f : X_1 \to X_2$ is continuous.

3. For $i = 1, 2, 3$, let $(X_i, \mathcal{T}_i)$ be a topological space. Let $f : X_1 \to X_2$ and $g : X_2 \to X_3$ be continuous maps. Prove that their composition $g \circ f : X_1 \to X_3$ is continuous.