1. Suppose (X_1, d_1) and (X_2, d_2) are metric spaces, and suppose $f : X_1 \to X_2$ is a function such that the preimage of every open set is open, i.e, for every open set $A_2 \subset X_2$, $f^{-1}(A_2)$ is open in X_1. Prove that f is continuous.

2. Find all topologies on the set $X = \{a, b, c\}$. In your list, identify the discrete and the indiscrete topologies.

3. (a) Let $X = \mathbb{R}$, $\mathcal{T} = \{[a, b] \mid a, b \in \mathbb{R}\} \cup \{\mathbb{R}, \phi\} \cup \{[a, \infty) \mid a \in \mathbb{R}\} \cup \{(-\infty, b] \mid b \in \mathbb{R}\}$. Is $\mathcal{T}$ a topology? Prove your answer.

   (b) Let $X = \mathbb{Z}$, the set of integers. Let $\mathcal{T}$ be the collection of all finite subsets of X. Is $\mathcal{T}$ a topology? Prove your answer.

4. Let (X, d) be a metric space. Prove that for every point $p \in X$, the set $\{p\}$ is closed in X.