1. Suppose $M_1 = (X, d_1)$ is a metric space. Let $M_2 = (X, d_2)$ be the metric space where $d_2 : X \times X \to [0, \infty)$ is defined by: $d_2(x, y) = 3d_1(x, y)$. Prove M_2 is a metric space.
2. Prove that an open ball in a metric space is an open set.
3. Prove that if A and B are open sets in a metric space, then $A \cap B$ is open.
4. Show by example that the intersection of an infinite collection of open sets is not necessarily open.
5. Prove that the union of any collection of open sets is an open set.
6. Use problems 2 and 5 to show that in any metric space, a set A is open iff it is a union of open balls.
7. Use problem 3 to prove that the union of two closed sets is closed. Hint: Use DeMorgan's Law, $(A \cap B)^c = A^c \cup B^c$.