Quiz 3. Math 460, Topology.
Instructor: Ramin Naimi

Name: ______________________
Mon 20 Oct 2003

Closed book. Closed Notes. 25 points per problem. Please write very legibly.

Do one problem from each of the following two sections. Extra Credit problems do not carry any points; so do not spend any time on them unless you're sure you've done your best with the problems that do carry points.

Section I.

1. Separate the following into classes of homeomorphic letters. No proofs or explanations necessary.

   A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

2. (a) True or false: If A and B are not disjoint, and each is a connected subspace of a topological space X, then $A \cap B$ is connected. Prove your answer.

   (b) True or false: If A and B are not disjoint, and each is a connected subspace of a topological space X, then $A \cup B$ is connected. Prove your answer.

3. Let $Y = S^1 \subset \mathbb{R}^2$. For each of the following spaces X, determine whether the map $f : X \to Y$ defined by $f(x) = (\cos(2\pi x), \sin(2\pi x))$ is a homeomorphism. Support your answers.

   (a) $X = (2, 3] \subset \mathbb{R}$.

   (b) $X = [3, 4] \subset \mathbb{R}$.

Section II.

4. (a) True or False: The union of two compact subsets of a topological space is compact. Prove your answer.

   (b) True or False: The union of infinitely many compact subsets of a topological space is compact. Prove your answer.

5. True or False: If $f : X \to Y$ is continuous and X is compact, then $f(X)$ is compact. Prove your answer.

Extra Credit Problems

6. (From Munkres's book, *Topology*, page 152.) A topological space is **totally disconnected** if its only connected subspaces are one-point sets. Show that if X has the discrete topology, then X is totally disconnected. Does the converse hold? Explain why or why not.

7. Let Y, X_1, and X_2 be topological spaces. For each $i = 1, 2$, let $f_i : Y \to X_i$ be a given map. Let $f : Y \to X_1 \times X_2$ be defined by $f(y) = (f_1(y), f_2(y))$.

   Prove f is continuous iff each of its component functions is continuous; i.e., f is continuous iff for each i, f_i is continuous.