Quiz 2. Math 460, Topology.
Instructor: Ramin Naimi

Name: ____________________
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Closed book. Closed Notes. 25 points per problem. Please write very legibly.

Extra Credit problems do not carry any points; so do not spend any time on them unless you're sure you've done your best with the problems that do carry points.

1. (a) (5 points) Give the definition of **homeomorphism**. Write complete and grammatically correct sentences.

   (b) (20 points) Prove $(2,5) \simeq \mathbb{R}$. Just find an appropriate map; you do not need to prove it's a homeomorphism. Give a brief explanation of how you come up with the map.

2. (a) (5 points) Give the definition of a **continuous** map between two metric spaces. Write complete and grammatically correct sentences.

   (b) (20 points) Suppose (X_1, d_1) and (X_2, d_2) are metric spaces, and suppose $f : X_1 \to X_2$ is a function such that the preimage of every open set is open, i.e, for every open set $A_2 \subseteq X_2$, $f^{-1}(A_2)$ is open in X_1. Prove that f is continuous according to the definition of continuity for metric spaces.

Extra Credit Problems

3. (No points) Recall the construction of the *Cantor Set:* Start with $[0,1] \subset \mathbb{R}$. Remove its open middle third, i.e., $(1/3, 2/3)$. You're left with $[0,1/3] \cup [2/3,1]$. Now remove the open middle third of each of the above two remaining closed intervals, i.e., remove $(1/9, 2/9)$ and $(7/9, 8/9)$. Keep repeating this process forever. What remains in the end is called the **Cantor Set**, which we denote as C.

   (a) Is C open, closed, both, or neither, in $\mathbb{R}$?

   (b) Prove that every point in C is a limit point of C. You may leave out tedious details and just give an outline of all the main ideas of the proof.

4. (No points) *Definition* Let (X,d) be a metric space. We say a sequence of points $a_1, a_2, a_3, \dots \in X$ **converges** to a point $p \in X$ if $\forall \epsilon > 0\ \exists M$ such that $\forall n > M$, $d(a_n, p) < \epsilon$. We write $\lim_{n\to\infty} a_n = p$, and say the sequence $a_1, a_2, a_3, \cdots$ is **convergent**.

   Let (X_1, d_1) and (X_2, d_2) be metric spaces. Prove that $f : X_1 \to X_2$ is continuous iff for every convergent sequence $a_1, a_2, a_3, \dots \in X_1$, $\lim_{n\to\infty} f(a_n) = f(\lim_{n\to\infty} a_n)$.