Quiz 1. Math 460, Topology.
Instructor: Ramin Naimi

Name: ______________________________
Fri 12 Sept 2003

Closed book. Closed Notes. 25 points per problem. Please write very legibly.

Do **only two** of the first four problems. The Extra Credit problems do not carry any points; so do not spend any time on them unless you're sure you've done your best with the problems that do carry points.

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1. (a) (5 points) Give the definition of an **open** subset of a metric space. Write a complete and grammatically correct sentence.
   (b) (20 points) True or False: the intersection of any collection of open subsets of a metric space is open. Prove your answer.
2. (a) (5 points) Give the definition of a **closed** subset of a metric space. Write a complete and grammatically correct sentence.
   (b) (20 points) True or False: in any metric space the intersection of any two closed sets is closed. Prove your answer. You may take as given that the union of any two open sets is open.
3. (a) (5 points) Give the definition of a **limit point**. Write a complete and grammatically correct sentence.
   (b) (20 points) For each of the following subsets of $\mathbb{R}$, give its interior, limit points, closure, and boundary. Just write your answer, without any explanations or proofs.
      i. $A = [1, 2) \cup (2, 3) \cup \{4\}$.
      ii. $A = \{1/n \mid n = 1, 2, 3, \cdots\}$.
4. (a) (5 points) Give the definition of an **open ball** in a metric space. Write a complete and grammatically correct sentence.
   (b) (20 points) Prove that in any metric space, every open ball is an open set.

Extra Credit Problems

5. (No points) Let A be a subset of a metric space M. Prove that $\overline{A}$ is closed in M.
6. (No points) Let A be a subset of a metric space M. Prove $\partial A = \{x \in M \mid \forall r > 0, B_r(x) \cap A \neq \emptyset \text{ and } B_r(x) \cap A^c \neq \emptyset\}$.