1. Draw two disjoint squares on a piece of paper and label their vertices, in clockwise order, $ABCD$ and $A'B'C'D'$. For example, in the first square, AB and AD are edges, and AC is a diagonal. For each of the identifications listed below, describe the resulting quotient space Q, as follows:

   (i) Is Q a manifold? (If not, explain why. And do not describe Q any further — skip to the next Q.)

   (ii) Is Q compact? What is the boundary of Q, if any?

   (iii) Is Q orientable?

   (iv) According to the classification of surfaces, Q is homeomorphic to one of S^2, nT^2, or $n\mathbb{R}\mathrm{P}^2$, minus some number (possibly zero) of disjoint open disks. Determine exactly which of these is the case (and the number of removed disks, if any).

   (a) $AB \sim A'B'$, $CD \sim C'D'$. (Note: orientations matter! For example, $AB \sim B'A'$ is not the same as $AB \sim A'B'$.)

   (b) $AB \sim A'B'$, $BC \sim B'C'$, $CD \sim C'D'$, $DA \sim D'A'$.

   (c) $AB \sim A'B'$, $CD \sim D'C'$.

   (d) $AB \sim A'B'$, $CD \sim AB$.

   (e) $AD \sim BC$, $A'D' \sim B'C'$, $AB \sim A'B'$, $CD \sim D'C'$.

   (f) $AB \sim A'B'$, $BC \sim B'C'$, $CD \sim C'D'$, $DA \sim A'D'$.

2. Draw an octagon $ABCDEFGH$ on a piece of paper. Let Q be the quotient space determined by the identifications $AB \sim DC$, $BC \sim ED$, $HA \sim GF$, $GH \sim FE$. Describe Q in terms of the same criteria as the previous problem.

3. Can the connected sum of two non-orientable surfaces be orientable? If yes, give an example. If not, explain your reasoning.

4. True or False: A surface is non-orientable iff it contains a Möbius band as a subset. Explain why (rigorous proof not necessary).

Extra Credit Problems

5. (a) Give a formal or informal definition of what it means for a 3-manifold to be orientable.

   (b) Can you think of any nonorientable 3-manifolds?