1. (a) Let P be the quotient space obtained by identifying opposite points on the unit circle; i.e., $P = \{S^1/(x, y) \sim (-x, -y)\}$. It turns out, surprisingly, that P is homeomorphic to S^1. Find a homeomorphism from P to S^1 (or vice versa). You don't have to prove that your map is a homeomorphism.

   (b) Let Q be the quotient space $\mathbb{R}/\{x \sim (x + 1)\}$. Which familiar topological space is Q homeomorphic to? Find a homeomorphism (without proof) to support your answer.

2. Let I^2 denote $[0, 1] \times [0, 1] \subset \mathbb{R}^2$.

   (a) Find an equivalence relation $\sim$ on I^2 such that the quotient space $I^2/\sim$ is homeomorphic to a cylinder, $S^1 \times I$.

   (b) Find an equivalence relation $\sim$ on I^2 such that the quotient space $I^2/\sim$ is homeomorphic to a torus, $S^1 \times S^1$.

3. Let $Q = \mathbb{R}^2/\sim$, where $\forall (x, y) \in \mathbb{R}^2$, $(x, y) \sim (x + 1, y) \sim (x, y + 1)$. Which familiar topological space is Q homeomorphic to? Find a homeomorphism (without proof) to support your answer.