1. Is the open unit ball in $\mathbb{R}^3$ compact? Prove your answer.

2. (a) Prove that the union of two compact subsets of a topological space is compact.
   (b) Prove that the union of infinitely many compact subsets of a topological space does not necessarily have to be compact.

3. Prove the following theorem: The continuous image of a compact set is compact.

4. *Definition* A subset A of a metric space X is **bounded** iff for some positive real number r and for some point $x \in X$, $A \subset B_r(x)$.

   Prove that every compact subset of a nonempty metric space is bounded.

   Hint: Let A be a compact subset of a metric space (X, d). Let x be an arbitrary point in X. Then $F = \{B_k(x) \mid k \in \mathbb{N}\}$ covers A (why?). Does F have a finite subcover?

5. *Definition* We say a function $f : X \to Y$, where X and Y are metric spaces, is **bounded** iff its image $f(X)$ is a bounded subset of Y.

   (a) Let I denote the closed unit interval, $[0, 1] \subset \mathbb{R}$. Prove that a continuous function $f : I \to \mathbb{R}$ cannot get arbitrarily large in magnitude; i.e, $f(I)$ must be bounded. (Hint: Use the Heine-Borel Theorem, and another theorem about the continuous image of compact sets.)
   (b) Give an example of a continuous function $f : (0, 1) \to \mathbb{R}$ that gets arbitrarily large in magnitude.

Extra Credit Problems

6. (From Munkres's book, *Topology*, page 152.) A topological space is **totally disconnected** if its only connected subspaces are one-point sets. Show that if X has the discrete topology, then X is totally disconnected. Does the converse hold?

7. Prove that the Cantor set, as a subspace of $\mathbb{R}$, is totally disconnected.