1. Prove that a space X is connected iff it contains no nonempty, proper subset which is both open and closed. (To say A is a **proper** subset of X means $A \subset X$ but $A \neq X$.)

2. True or false: If A and B are not disjoint, and each is a connected subspace of a topological space X, then $A \cap B$ is connected. Prove your answer.

3. True or false: If A and B are not disjoint, and each is a connected subspace of a topological space X, then $A \cup B$ is connected. Prove your answer.

4. (a) Prove the following theorem:
   Theorem: The continuous image of a connected set is connected; i.e, if $f : X \to Y$ is a continuous map between topological spaces, and if X is connected, then f(X) is connected.

   (b) Prove the following corollary:
   Corollary: If X is connected, and Y is homeomorphic to X, then Y is connected.

Extra Credit Problems

5. Prove the following theorem: $A \subset \mathbb{R}$ is connected iff A is an interval (open, closed, or half open; infinite or half-infinite).