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1. Let $f : \mathbb{R} \to \mathbb{R}$ be given by: $f(x) = \begin{cases} 1/2 & \text{if } x < 0 \\ 1/3 & \text{if } x \geq 0 \end{cases}$. Prove that f is not continuous at 0.

2. In the following, just find a map each problem asks for, without proving continuity, injectivity, or surjectivity. Each of the following sets is assumed to come with the standard Euclidean metric.

   (a) Let $M_1 \subset \mathbb{R}^2$ be the closed unit disk (i.e., $M_1 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$). Let $M_2 \subset \mathbb{R}^2$ be the closed disk of radius 2 centered at the origin. Find a continuous bijection (one-to-one and onto map) $f : M_1 \to M_2$.

   (b) Let $M_3 \subset \mathbb{R}^2$ be the closed disk of radius 1 centered at the point $(3, 4)$. Find a continuous bijection $f : M_1 \to M_3$.

   (c) Let $M_4 \subset \mathbb{R}^2$ be the closed disk of radius 2 centered at the point $(3, 4)$. Find a continuous bijection $f : M_1 \to M_4$.

3. Suppose $M_1 = (X_1, d_1)$ and $M_2 = (X_2, d_2)$ are metric spaces. Let $b \in X_2$, and let $f : X_1 \to X_2$ be the constant map $f(x) = b$, $\forall x \in X_1$. Show that f is continuous on X_1.

4. Suppose $M_1 = (X_1, d_1)$ and $M_2 = (X_2, d_2)$ are metric spaces, and suppose $f : X_1 \to X_2$ is a continuous function. Prove that $\forall a \in X_1$ and $\forall \epsilon > 0$, $\exists \delta > 0$ such that the ball of radius δ around a is mapped under f to inside the ball of radius ϵ around $f(a)$; i.e., $f(B_\delta(a)) \subseteq B_\epsilon(f(a))$.

5. Suppose $M_1 = (X_1, d_1)$ and $M_2 = (X_2, d_2)$ are metric spaces, and suppose $f : X_1 \to X_2$ is a continuous function. Prove that the preimage of any open set in M_2 is an open set in M_1; i.e., if $A_2 \subset X_2$ is open, then $A_1 = f^{-1}(A_2) \subset X_1$ is also open.

Extra Credit Problems

6. The *Cantor Set.*

   Start with $[0, 1] \subset \mathbb{R}$. Remove its open middle third, i.e., $(1/3, 2/3)$. You're left with $[0, 1/3] \cup [2/3, 1]$. Now remove the open middle third of each of the above two remaining closed intervals, i.e., remove $(1/9, 2/9)$ and $(7/9, 8/9)$.

   What's remaining now? What is the open middle third of each remaining piece?

   Keep repeating this process forever. What remains in the end is called the **Cantor Set**, which we denote as C.

   Is C open, closed, both, or neither, in $\mathbb{R}$?

   Prove that every point in C is a limit point of C.