

- State which of the following spaces are homeomorphic to each other. You do not need to prove your answers; but give brief explanations or draw pictures (or both) to support them.
 - $X = \overline{B_1(0,0)} \subset \mathbb{R}^2$ (i.e., X is the closed unit disk in the plane).
 - $Y = X - B_{0.5}(0,0)$.
 - Y° (the interior of Y , where Y is viewed as a subspace of \mathbb{R}^2).
 - $X/\partial X$. (This means identify the whole boundary of X into one point – recall that $\partial X = S^1$.)
 - $S^1 \times [0, 1]$.
 - $S^1 \times (0, 1)$.
 - $S^1 \times [0, 1)$.
 - $S^1 \times \mathbb{R}$.
 - \mathbb{R}^2
 - $S^2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$.
 - $S^2 - \{(1, 0, 0)\}$.
 - $S^2 - \{(0, 0, 1), (0, 0, -1)\}$.
 - $S^2 - B_{0.5}(0, 0, 1)$.
- Which of the following letters, considered as subspaces of \mathbb{R}^2 , are manifolds? Give a brief explanation for each one that is not a manifold.

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- How many non-homeomorphic 1-manifolds do you think there are? List as many as you can think of.
- Is an open ball minus its center a manifold? More precisely, let $x \in \mathbb{R}^n$, $r > 0$. Is $B_r(x) - \{x\}$ a manifold? Prove your answer.
- Is $S^1/\{(1, 0) \sim (-1, 0)\}$ a manifold? Explain.
 - Is $S^1/\{(x, y) \sim (-x, -y)\}$ a manifold? Explain.
- Give an example of topological spaces X and Y such that X is locally homeomorphic to Y but Y is not locally homeomorphic to X .

Extra Credit Problems

- Definition* A **component** of a topological space X (sometimes also called a *connected component*, for emphasis) is a connected subset of X that is not a subset of any other connected subset of X (in short, it's a maximal connected subset of X).
Let X be a topological space. Prove that every component of X is open and closed. (Hint: Let C be a component of X . Prove C is the union of all open connected subsets of X that intersect C .)
- Definition* A topological space X is path connected if $\forall x, y \in X$ there is a **path** from x to y ; i.e., there is a continuous map $f : [0, 1] \rightarrow X$ such that $f(0) = x$ and $f(1) = y$.
Prove whether each of the following is true or false. (You may give a sketch of the proof, containing all the necessary ingredients, but perhaps missing some of the tedious details.)
 - Every connected space is path connected.
 - Every path connected space is connected.

Hint: For one of the above, consider the following subset $X \subset \mathbb{R}^2$, with the subspace topology inherited from \mathbb{R}^2 , called the *topologist's sine curve*.

$$X = \{(x, y) \in \mathbb{R}^2 \mid (x = 0 \text{ and } -1 \leq y \leq 1) \text{ or } (x > 0 \text{ and } y = \sin(1/x))\}$$