1. Prove that a space X is connected iff it contains no nonempty proper subset which is both open and closed. (A is a **proper** subset of X iff $A \subset X$ but $A \neq X$.)

2. True or false: If A and B are connected subspaces of a topological space X and $A \cap B \neq \emptyset$, then $A \cap B$ is connected. Prove your answer.

3. True or false: If A and B are connected subspaces of a topological space X and $A \cap B \neq \emptyset$, then $A \cup B$ is connected. Prove your answer.

4. Suppose $f : X \to Y$ is continuous, and $A \subset f(X)$ is open in $f(X)$, where $f(X)$ inherits the subspace topology from Y. Prove that $f^{-1}(A) \cap X$ is open in X.

5. Prove the following.

   (a) Theorem: The continuous image of a connected set is connected; i.e, if $f : X \to Y$ is a continuous map between topological spaces and X is connected, then $f(X)$ is connected.

   (b) Corollary: If X is connected, and Y is homeomorphic to X, then Y is connected.

Extra Credit Problems

6. Prove the following theorem: $A \subset \mathbb{R}$ is connected iff A is an interval (open, closed, or half open; infinite or half-infinite).