1. Suppose (X_1, d_1) and (X_2, d_2) are metric spaces, and suppose $f : X_1 \to X_2$ is a function such that the preimage of every open set is open, i.e, for every open set $A_2 \subset X_2$, $f^{-1}(A_2)$ is open in X_1. Prove that f is continuous according to the definition of continuity for metric spaces.

2. Find all topologies on the set $X = \{a, b, c\}$. Just list the different topologies, without proving that they really are topologies. In your list, identify the discrete and the indiscrete topologies.

3. Let $X = \mathbb{R}$. Let $A = \{[a, b] : a, b \in \mathbb{R}, a < b\} \cup \{[a, \infty) : a \in \mathbb{R}\} \cup \{(-\infty, b] : b \in \mathbb{R}\}$. Let $\mathcal{T}$ consist of all finite unions of sets in A, together with $\emptyset$. Is $\mathcal{T}$ a topology? Prove your answer.

4. Let (X, d) be a metric space. Prove that for every point $p \in X$, the set $\{p\}$ is closed in X.

Extra Credit Problems

5. Find a homeomorphism between the Cantor set C (see previous homework for definition) and its left half, i.e., $C \cap [0, 1/3]$.