1. Prove that if X and Y are homeomorphic topological spaces, then X is simply connected iff Y is.
2. Prove or disprove: $\mathbb{R}$ is simply connected.
3. Prove or disprove: Let $\sim$ be an equivalence relation on a topological space X. If X is path connected, then the quotient space $X/\sim$ is path connected.
4. Prove or disprove: Let $\sim$ be an equivalence relation on a topological space X. If X is simply connected, then the quotient space $X/\sim$ is simply connected.
5. Prove or disprove: If A and B are non-disjoint path connected subspaces of a topological space X, then $A \cap B$ is path connected.
6. Prove or disprove: If A and B are non-disjoint path connected subspaces of a topological space X, then $A \cup B$ is path connected.
7. Prove or disprove: If A and B are simply connected subspaces of a topological space X such that $A \cap B$ is path connected, then $A \cap B$ is simply connected.
8. Let X_1 and X_2 be simply connected topological spaces. Prove that $X_1 \times X_2$ is simply connected. Hint: Let $f : I \to X_1 \times X_2$ be a loop. For $i = 1, 2$, let $\pi_i : X_1 \times X_2 \to X_i$ be the projection map. What can you say about the map $\pi_i \circ f : I \to X_i$?

Extra Credit Problems

9. Prove or disprove: If A and B are simply connected subspaces of a topological space X such that $A \cap B$ is path connected, then $A \cup B$ is simply connected.