1. For each of the following manifolds, state without proof (i) its dimension; (ii) its boundary (if any); (iii) whether or not it's compact; (iv) whether or not it's a closed manifold.
   (a) $X = \overline{B_1(0,0)} \subset \mathbb{R}^2$ (i.e., X is the closed unit disk in the plane).
   (b) $Y = X - B_{0.5}(0,0)$. (c) Y° (the interior of Y, where Y is viewed as a subspace or $\mathbb{R}^2$).
   (d) $Z = X/\partial X$. (This means identify the whole boundary of X into one point – recall that $\partial X = S^1$.)
   (e) $S^1 \times [0,1]$ (f) $S^1 \times (0,1)$ (g) $S^1 \times [0,1)$ (h) $S^1 \times \mathbb{R}$ (i) $\mathbb{R}^2$
   (j) $S^2 = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 = 1\}$ (k) $S^2 - \{(0,0,1)\}$ (l) $S^2 - \{(0,0,1),(0,0,-1)\}$
   (m) $S^2 - B_{0.5}(0,0,1)$ (n) $S^2 \times [0,1]$ (o) $B_1(0,0,0)$ (p) $\mathbb{R}^3_+$
   (q) $B_1(0,0,0) - B_{0.5}(0,0,0)$ (r) $S^2 \times [0,1)$ (s) $T^2 \times [0,1]$

2. State, without proof, whether or not each of the following topological spaces is a manifold (with or without boundary). If you claim that it is a manifold, give its dimension and boundary, and state whether or not it is closed. If you claim that it is not a manifold, give a brief reason.
   (a) $\mathbb{R}^2_+ - \{(0,0)\}$ (b) $\mathbb{R}^2_+ - \{(x,y) \mid -1 \leq x < 1,\ y = 0\}$ (c) $\mathbb{R}/\{x \sim -x\}$
   (d) $[B_2(0,0) \cup B_2(5,0)]/\{\forall (x,y) \in B_1(0,0),\ (x,y) \sim (x+5,y)\}$
   (e) $[B_2(0,0) \cup B_2(5,0)]/\{\forall (x,y) \in \overline{B_1(0,0)},\ (x,y) \sim (x+5,y)\}$

3. Suppose X is a discrete topological space (i.e. it has the discrete topology) with at least two points. Prove that X is not connected.

4. Let $X = \overline{B_1(0,0)} - B_{0.5}(0,0)$, $Y = S^1 \times [0,1]$. Find a homeomorphism $f : X \to Y$. (You need not prove your map is a homeomorphism.)

Extra Credit Problems

5. Prove that every compact 3-manifold $M \subset \mathbb{R}^3$ has boundary.

6. (a) Find an example of a collection $B_1 \supseteq B_2 \supseteq B_3 \supseteq \cdots$ of nested nonempty closed subsets of $\mathbb{R}$ whose intersection $\bigcap B_i$ is empty.
   (b) Let $B_1 \supseteq B_2 \supseteq B_3 \supseteq \cdots$ be a collection of nested nonempty closed subsets of a compact topological space X. Prove their intersection $\bigcap B_i$ is nonempty.